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# SIMPLIFYING ROBUST CONTROL DESIGNS OF PARAMETRIC UNCERTAIN SYSTEMS USING DIMENSIONAL TRANSFORMATIONS

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## ABSTRACT

Robust control techniques have become more popular in the last decade because of their ability to construct a single controller suitable for a family of plants that are represented as a nominal plant with an uncertainty bounds. A typical challenge with robust control design is that, as the size of the uncertainty block gets bigger, the complexity of the control synthesis and resulting controller exponentially increases. Using a dimensional transformation method, this work shows that some robust control problems can be recast into an equivalent representation with a much smaller size of uncertainty block. This reduction potentially reduces the conservativeness and computational complexity of both $H_\infty/\mu-$ synthesis methods. This method is demonstrated using the 1990 ACC benchmark problem.

## INTRODUCTION

Classical and modern control techniques that are based on linear control design methodologies are today quite mature and mathematically understood. Despite this understanding, linear controllers still often do not meet the required performance criteria in practice even though their theoretical design and evaluation shows otherwise. Robust control design is based on the idea that an uncertain plant can be described as a nominal system along with a functional description of uncertainty. Uncertainty includes such dynamics as nonlinearities, immeasurable noise, parametric uncertainties or modeling errors, etc., which are generally grouped into two general types [1, 2]: *Parametric uncertainty* and *Unmodelled dynamics uncertainty*. The core of the robust control design paradigm is based on notion of a family of plants which are generally represented as a nominal linear plant and all perturbations from the nominal, grouped as an uncertainty set. The robust controller synthesis problem seeks to find a single controller that is guaranteed to stabilize all the plants in the family. A key procedure to synthesize robust controllers is the $H_\infty$ control synthesis based on small gain theorem, introduced by Zames [3]. In this technique, the problem of $H_\infty$ control is cast as an optimization problem subject to constraints in the frequency domain.

Since then, a very significant body of research has developed advanced techniques of analysis and synthesis of a robust control system [4-10] with many focusing on simplifying uncertainty or plant descriptions. Fu [4] has shown that the robust stability problem with both parametric and nonparametric uncertainties can be unified into a single robust stability problem with parametric uncertainty only. In [5], Sideris showed that robust performance in linear feedback systems with: 1) parametric model uncertainty and 2) robust stability requirements under combined parametric model uncertainty and unmodelled dynamics are reduced to an equivalent single problem of analyzing robust stability with respect to uncertain parameters. In 1982 both Doyle [6] and Safonov [7] independently developed a robustness measure that takes into account the structure of an uncertainty description of many practical problems. The former introduced the structured singular value $-\mu$. The robust control synthesis procedure that is based on this structured singular value is called $\mu-$synthesis. In [8-10] the authors mention a problem of discontinuity when dealing with real or mixed real/complex parametric uncertainties, while many have presented methods and theoretical analysis to overcome this problem, research in this area remains strong.

 

The reason so many have focused on simplification of the uncertainty representation is because problem complexity and/conservativeness increases with size of uncertainty block. Fan et al. [11] discussed the exponential increase in computational complexity with the number of uncertain parameters, and that this limits the "usefulness of exhaustive global search methods" in control synthesis. Current computation methods of the structured singular value, $\mu$, are often limited to calculating its upper and lower bounds. For uncertainty block size $\leq 3$, the actual $\mu$ can be computed [11-13]. However, for block size > $3$, $\mu$ can't be computed exactly, and the gap between the upper and lower bounds can be arbitrarily large resulting in a more conservative controller synthesis [11, 13].

Current robust controller design methods require the formulation of the plant and uncertainty descriptions in a form called linear fractional transformation (LFT) as shown in Fig. 1. This form, usually represented in short hand as $P-\Delta$, is a separate system description of the nominal input-output system $P$ and the uncertain input-output system $\Delta$. For practical problems, the formulation of a $P-\Delta$ model that accurately characterizes realistic system uncertainties is critical because the robustness results of any controller design depends directly

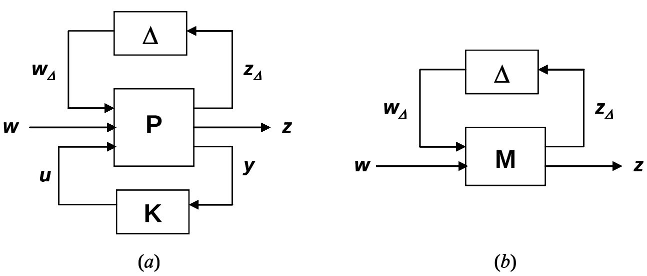


**Fig. 1:** A common robust control setup. (a) Interconnection structure of a general uncertain closed-loop system (b) LFT uncertainty description

on the uncertainty model used in the analysis or design. An overview of such modeling is presented in Belcastro [14]. In Fig. 1(*b*) assume that the system $M(s)$ is nominally stable and the perturbations $\Delta(s)$ are stable. If these conditions are satisfied, then the $M-\Delta$ structure is stable [1] for all perturbations $\Delta$ satisfying $\|\Delta\|_\infty \leq 1$ if and only if:

$$\bar{\sigma}(M(j\omega))<1 \quad \forall\omega \quad \Leftrightarrow \quad \|M\|_\infty <1 \tag{1}$$

The current work focuses on the reformulation of the parametric uncertainty model in order to reduce the size of the parametric uncertainty block. By reducing the size of the parametric block, the goal is to allow larger allowable perturbation in both the unstructured and structured uncertainty block descriptions.

The paper is organized as follows: A compact method of dimensional analysis is presented first, then developed via an example control synthesis designed for the 1990 ACC benchmark problem. Simulation results are presented that show that the dimensionless system representation with reduced parametric block size has better performance margins than the classical representation. In following sections, limitations of this approach are also discussed especially with regard to implementing the dimensionless controllers under current technology. These limitations are currently under study and outlined in the end of the paper. Finally the main points are summarized in the Summary section.

## A MOTIVATING EXAMPLE

The 1990 ACC benchmark problem [15] is presented to illustrate the main points of this work. The system consists of a coupled two-mass and a spring system without damping and is non-collocated, as shown in Fig. 2. This system is commonly used in the robust control community.

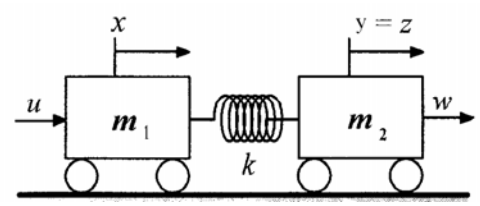


**Fig. 2**: The 1990 ACC benchmark problem

The equation of motion of the system shown in Fig. 2 is given by Eq. (1).

$$\begin{aligned} m_1\ddot{x}+kx-ky&=u \\ m_2\ddot{y}-kx+ky&=w \end{aligned} \tag{2}$$

All the parameters in Eq. (1) are uncertain, i.e., $m_1=m_{10}+\delta_{m1}$ , $m_2=m_{20}+\delta_{m2}$ , $k=k_0+\delta_k$ , where $\delta_i$'s are the perturbations from the nominal values and are assumed to be bounded, for instance in this example, it's assumed that $\|\delta_i\|\leq 0.2$.

## MODELING OF THE PARAMETRIC UNCERTAIN SYSTEM

The system of Eq. (2) is represented in the classical form or parametric uncertainty form as shown in Fig. 3(*a*) and Fig. 3(*b*), respectively. However, for robust control synthesis, the uncertainty model representation is used because it is easy to convert it to the LFT form of the system. As can be seen from Fig. 3(b), the system has a diagonal uncertainty block of size 3 as given by Eq. (3).

$$\begin{bmatrix} w_{m2} \\ w_{m1} \\ w_k \end{bmatrix} = \begin{bmatrix} \delta_{m2} & 0 & 0 \\ 0 & \delta_{m1} & 0 \\ 0 & 0 & \delta_k \end{bmatrix} \begin{bmatrix} z_{m2} \\ z_{m1} \\ z_k \end{bmatrix} \tag{3}$$

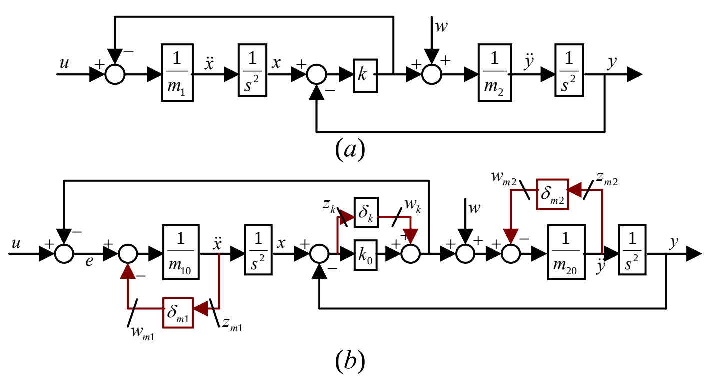


**Fig. 3:** (a) classical model, (b) uncertainty model

## THE METHOD OF DIMENSIONAL TRANSFORMATION AND VARIABLE REDUCTION

In this section, the process of dimensional transformation is generalized and presented, with the above example used to illustrate the mechanics of the transformation process. To generalize dimensional analysis to system theory, consider a functional relation a actual physical system as $f(x_1,...x_{N_x}, p_1,..., p_{N_p}) = 0$, where the $x_i$'s and $p_i$'s are signals and parameters as indicated in Tab. 1. In this paper the term signals refers to state variables, input and output variables and time where as all other variables used in the dynamic description of the system are called parameters. The Buckingham-Pi theorem [16] proves that this equation can also be represented in an information-equivalent form as $\bar{f}(\pi_1, \pi_2,..., \pi_{N_\pi}) = 0$, where the $\pi_i$'s are formed by grouping of the $x_i$'s and $p_i$'s and are dimensionless. The Pi-Theorem states that $N_\pi \le N$, where $N_\pi$ is the number of variables in the dimensionless formulation (usually called $\pi$ variables), and $N$ is the number of variables in the original formulation. In summary, there are often less variables required to express the equations of a physical law, for instance equations of motion, with dimensionless parameters than when dimensioned representations are used.

To formalize the process of a dimensional transformation, consider any functional description of dynamic behavior (a plant or controller) dependent on $N$ variables. If the description is a dynamic one, the variables will generally span unit dimensions of length, mass, and distance. Hereafter we assume that the units of each variable $v$ can be written as a vector that is extracted via a dimensional extraction operator, $d_{v,e} = D(e,v)$. To uniquely define this vector, one must specify both the unit space as well as the parameter. For instance, the gravitational constant, $g = 9.81\ \mathrm{m/s^2}$, has dimensional units that can be represented in one unit system, $e = [m \quad kg \quad s]^T$, as a column vector, $d_{g,e} = D(e,g) = [1 \quad 0 \quad -2]^T$, or in another unit system of $e = [kg \quad N]^T$, as $d_{g,e} = D(e,g) = [-1 \quad 1]^T$.

The dimensional unit system is known to be an arbitrary factor in representing a system [17], therefore we seek to rescale the system by selecting unit systems that give specific advantage to the robust control problem, e.g. ones that most simplify the uncertainty representation. Some basis systems are clearly advantageous for controller design purposes, particularly ones producing dimensionless representations (see Brennan [18]). The transformation from/to a dimensioned to/from a dimensionless system is fairly straightforward and follows a simple basis transformation (see Szirtes [19]) which are in turn formalizations of unit normalization procedure first described in the Buckingham-Pi Theorem [19] nearly a century ago.

First, one notes the vector representing the units of each parameter or signal, hereafter called a variable, using the $d_{v,e} = D(e,v)$ operation. One then writes the resulting vectors as column of the matrices $B_D$ or $A_D$, in Tab. 1. For clarity, the variable labels are shown at top of the matrices and unit dimensions are shown at left. In the case of Tab. 1, the general unit system is selected as, $u = [e_1 \quad e_2 \quad \dots \quad e_{N_e}]^T$. For example, in Tab. 1, if $p_1 = g$ (the gravitational constant), then for a unit system $u = [e_1 \quad e_2 \quad e_3]^T = [m \quad kg \quad s]^T$, the $N_x+1$ column of $B_D$ would be $d_i = [1 \quad 0 \quad -2]^T$. If $p_{N_p} = g$ instead**,** then the last column of $A_D$ would be $d_i = [1 \quad 0 \quad -2]^T$, and so on. This matrix representation $[B_D \ A_D]$ will always have $N_e$ rows and $N_x + N_p$ columns. In this process, variables forming columns in matrix $A_D$ are considered the scaling variables whereas those forming columns in matrix $B_D$ are called scaled variables.

| | signals: $x_1$ $x_2$ $\cdots$ $x_{N_x}$ / parameters: $p_1$ $p_2$ $\cdots$ | parameters: $\cdots$ $p_{N_p-1}$ $p_{N_p}$ |
|---|---|---|
| $e_1$, $e_2$, $\vdots$, $e_{N_e}$ | $B_D$ | $A_D$ |
| $\pi_1$, $\pi_2$, $\vdots$, $\pi_{N_\pi}$ | $I$ | $C_S$ |

**Tab. 1:** The dimensional transformation process

The challenge of dimensional scaling is to choose variables to participate in the scaling matrix $A_D$ such that the transformed system is most amenable to robust control. To choose scaling variables, one rearranges the columns of the matrices $A_D$ and $B_D$, selecting $N_e$ variable-columns for $A_D$ such that the corresponding columns are linearly independent, i.e. they can together form the full rank matrix $A_D$ (rank = $N_e$**)**. The requirement for a full rank matrix $A_D$ is always feasible if the unit system is not redundant (see Szirtes [19]). From a dimensional analysis viewpoint, the choice of which variables to place in $A_D$ is a user-defined choice with the *only* restriction being that $A_D$ has to be full rank. From a control theory standpoint, however, the user should avoid placing signals in this matrix. The variables in the scaling matrix will be multiplied and divided through the variables of $B_D$ such that dimensionless parameters are produced. The use of a signal may cause a division by zero if a signal used to scale other variables instantaneously attains a zero value.

For convenience, we hereafter assume that the matrices $A_D$ and $B_D$ are arranged such that the $N_e$ columns of $A_D$ occur on the $N_e$ right-most columns. This allows the partitioning of the matrix in the form shown in Tab. 1, where $e_i$'s are the dimensional units that span the $x_i$'s and $p_i$'s. To represent the variables of $B_D$ in the new dimensional basis given by the column vectors associated with $A_D$, one calculates the matrix:

 

$$C_S = \left(-A_D^{-1}.B_D\right)^T \tag{4}$$

The lower left partition of Tab. 1 is always unity to generate dimensionless parameters.

This process of dimensional transformation is demonstrated on the ACC benchmark problem, and results are presented in Tab. 2, where the scaling parameters are chosen to be $m_1$, $k$ and $g$. The bottom-left rows of the partitioned matrix in Tab. 2 indicate the dimensionless variables (signals and parameters) of the system, with the number of dimensionless variables, $N_\pi$, given by $N_\pi = N_x + N_p - N_e$, and is equal to 6 in the ACC benchmark example (by inspection).

| | $t$ | $x$ | $y$ | $u$ | $w$ | $m_2$ | $m_1$ | $k$ | $g$ |
|---|---|---|---|---|---|---|---|---|---|
| $m$ | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |
| $kg$ | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| $s$ | 1 | 0 | 0 | −2 | −2 | 0 | 0 | −2 | −2 |
| $\pi_1$ | 1 | 0 | 0 | 0 | 0 | 0 | −1/2 | 1/2 | 0 |
| $\pi_2$ | 0 | 1 | 0 | 0 | 0 | 0 | −1 | 1 | −1 |
| $\pi_3$ | 0 | 0 | 1 | 0 | 0 | 0 | −1 | 1 | −1 |
| $\pi_4$ | 0 | 0 | 0 | 1 | 0 | 0 | −1 | 0 | −1 |
| $\pi_5$ | 0 | 0 | 0 | 0 | 1 | 0 | −1 | 0 | −1 |
| $\pi_6$ | 0 | 0 | 0 | 0 | 0 | 1 | −1 | 0 | 0 |

**Tab. 2:** The matrices used for the dimensional transformation process

Each bottom row indicates how new dimensionless variables, hereafter called $\pi$-variables, should be created from powers of each of the column variables of $A_D$. As a specific example, the first row gives the first $\pi$-variable: $\pi_1 = t \cdot m_1^{-1/2} \cdot k^{1/2}$. All the $\pi$-variables (scaled signals and parameters) of the ACC benchmark problem are shown in Eq. (5).

$$\pi_1 = t\sqrt{k/m_1} = \tau,\quad \pi_2 = \frac{kx}{m_1 g} = \bar{x},\quad \pi_3 = \frac{ky}{m_1 g} = \bar{y},$$
$$\pi_4 = \frac{u}{m_1 g} = \bar{u},\quad \pi_5 = \frac{w}{m_1 g} = \bar{w},\quad \pi_6 = \frac{m_2}{m_1} \tag{5}$$

Note that the first five $\pi$ variables are associated with signals, and the last one with parameters. The original system had three uncertain parameters, while clearly the dimensionless representation has only one.

## THE DIMENSIONLESS REPRESENTATION OF THE SYSTEM

Based on the transformation given Eq. (3), the system equation (Eq. (2)) in dimensionless form is given by Eq. (6),

$$\begin{aligned} &\bar{x}'' + \bar{x} - \bar{y} = \bar{u} \\ &\pi_6 \bar{y}'' - \bar{x} + \bar{y} = \bar{w} \end{aligned} \tag{6}$$

Where, $(') \equiv \dfrac{d}{d\tau} = \sqrt{m_1/k}\,\dfrac{d}{dt}$, and the bars on the top of the variables represent the dimensionless (normalized) representation of the variables. Fig. 4 is the dimensionless representation of the system shown earlier in Fig. 3. As can be seen in Fig. 4(b), the system has only one uncertain parameter. Hence the parametric uncertainty block size has been reduced from three in the dimensional system to only one in the dimensionless system as a result of the dimensional transformation.

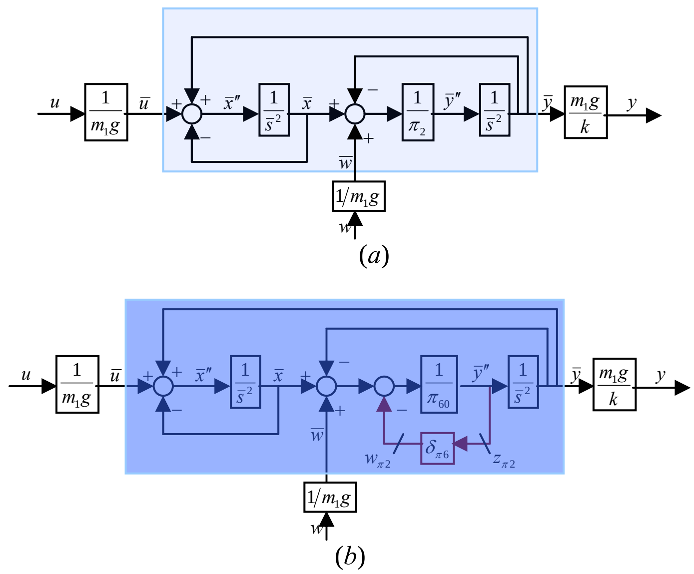


**Fig. 4:** Model in Dimensionless form: (a) classical model, (b) uncertainty model

The benefit of this uncertainty reduction is explored through a comparison of the numerical results of the two system representation in the next subsection.

## SIMULATION RESULTS

To show a comparison in performance of the two system representations, controllers are synthesized for each using the $H_\infty$-synthesis toolbox of MATLAB. The performance specs are chosen to be the same for both cases, i.e., 1) robust stability for all values of $|\delta_i| \le 0.2$, 2) for the nominal system in response to an impulse disturbance acting at the second mass, the peak control action and the settling time on the displacement of the second mass should satisfy $|u| \le 1$ and $t_s \sim 15$ sec, respectively. The nominal parameters used are: $m_{10} = m_{20} = k = 1.0$. The problem is cast as robust stability/performance problem and the uncertainty description for both the dimensional and dimensionless descriptions are given by Eq. (7) and Eq. (8) respectively.

$$\begin{bmatrix} w \\ w_{m2} \\ w_{m1} \\ w_k \end{bmatrix} = \begin{bmatrix} \delta_u & \delta_y & 0 & 0 & 0 \\ 0 & 0 & \delta_{m2} & 0 & 0 \\ 0 & 0 & 0 & \delta_{m1} & 0 \\ 0 & 0 & 0 & 0 & \delta_k \end{bmatrix} \begin{bmatrix} u \\ z \\ z_{m2} \\ z_{m1} \\ z_k \end{bmatrix} \tag{7}$$

$$\begin{bmatrix} \overline{w} \\ \overline{w}_{\pi 6} \end{bmatrix} = \begin{bmatrix} \delta_{\overline{u}} & \delta_{\overline{y}} & 0 \\ 0 & 0 & \delta_{\pi 6} \end{bmatrix} \begin{bmatrix} \overline{u} \\ \overline{z} \\ \overline{z}_{\pi 6} \end{bmatrix} \tag{8}$$

The nominal results show that the performance specs were better achieved when using the dimensionless representation compared to the dimensional representation. The results of simulation are given below (Fig. 5 and Fig. 6) and the main differences are summarized in Tab. 3. The first difference between the two results is that the dimensionless system has better performance: such as lower overshoot (by about 20%), and lower control effort (by about 25%). Note that the maximum allowable $H_\infty$ norm for the two representations is different. For the dimensional case, it is directly evaluated as $|\delta_i|^{-1} = 5.0$. For the dimensionless case, first the uncertainty bound of $\pi_6$ is evaluated from the uncertainty bounds of $m_1$ and $m_2$. Using the definition of $\pi_6$ in Eq. (5), the minimum, nominal and maximum values of $\pi_6$ are defined in Eq. (9),

$$\begin{aligned} \pi_{6\min} &= m_{1\min} / m_{2\max} = 0.8/1.2 = 0.67 \\ \pi_{60} &= m_{10} / m_{20} = 1.0/1.0 = 1.0 \\ \pi_{6\max} &= m_{1\max} / m_{2\min} = 1.2/0.8 = 1.5 \end{aligned} \tag{9}$$

from which the uncertainty bound and the allowable $H_\infty$ norm can be defined, respectively as: $|\delta_{\pi 6}| = 0.5$ and $|\delta_{\pi 6}|^{-1} = 2.0$.

| | Dimensional representation | Dimensionless representation | Comparison |
|---|---|---|---|
| $H_\infty$ norm | 4.977 < 5.0 | 1.576 < 2.0 | Better margin: 1.01 vs 1.27 |
| overshoot | 2.2 | 1.8 | Improved overshoot by ~ 20% |
| Control action | 0.95 | 0.7 | Less control effort by ~ 25% |

**Tab. 3:** Summary of the simulation results

Perhaps another most important difference between the two systems is the stability margin. This can clearly be deducted from the fact that the dimensionless representation has more allowable perturbation (*margin* $\cong$ *1.27*) compared to the dimensional representation (*margin* $\cong$ *1.01*).

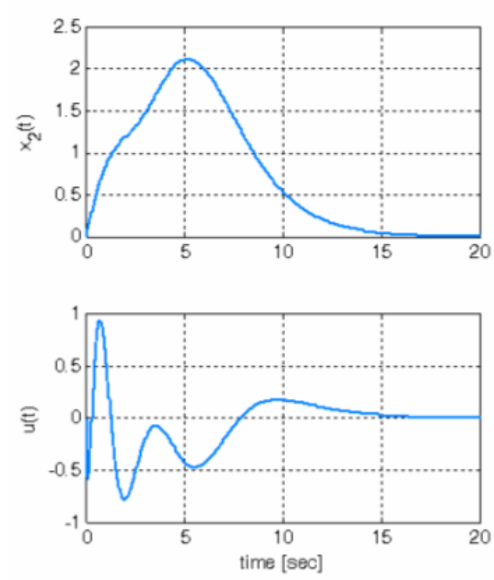


**Fig. 5:** Dimensional system, $H_\infty$ *norm 4.977 < 5.0, (*$|\delta_i|$*<0.2)*

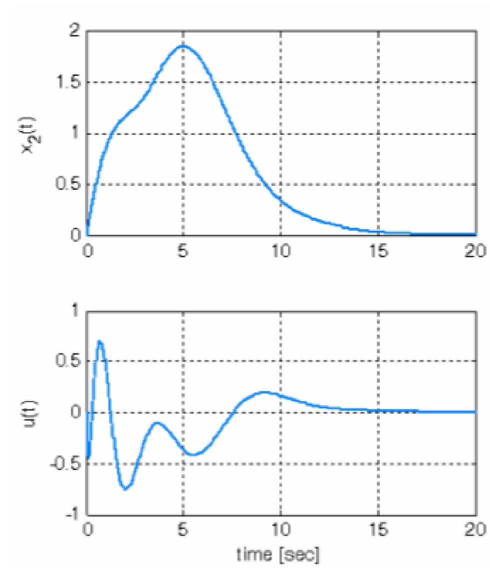


**Fig. 6:** Dimensionless system, $H_\infty$ *norm 1.576 < 2.0, (*$|\delta_{\pi 6}|$*<0.5).*

## LIMITATION OF THE APPROACH

The presented method of parametric uncertainty reduction has two potential limitations. The first is that the dimensionless uncertainty description may have wider uncertainty bound than the dimensional case. For example, in the ACC benchmark problem considered above, the individual parametric uncertainty bounded as $|\delta_i| \le 0.2$. However, after the dimensional transformation, the dimensionless representation has one parametric uncertainty that must be bounded as $|\delta_{\pi 6}| \le 0.5$ to capture the equivalent parametric variation from the original representation. Therefore there is a tradeoff between reducing the number of uncertainty parameters and widening of the uncertainty bounds. This may play a negative role in trying to reduce the conservativeness of the problem.

The second limitation is in regard to the implementation of robust controller designed in the dimensionless domain. This is because the controller cannot be directly implemented as physical systems are dimensioned. In other words, the output from the real plant has to be transformed into a dimensionless signal to be used as feedback into the dimensionless controller. The output of the dimensionless controller needs to be transformed into dimensioned signal since it is used to actuate the plant. In both transformations, the signals may be scaled by uncertain parameters. There are two scenarios: 1) The input to the controller (sensor signal) and the output share the same physical units (such as Force-Force, displacement-displacement, torque-work, etc…), any uncertainty on the input-output scaling of the controller cancels each other and there would be no difficulty in the implementation of the controller. 2) The input-output of the controller are not the same physical quantities (such as displacement-force, displacement-pressure, etc…) then there may be an uncertainty to the input-output scaling that would therefore make the output of the controller uncertain.

The issue with uncertain scaling, the second scenario discussed above, may be mitigated by re-inclusion of some of the uncertainty back into the design model. The re-inclusion of this uncertainty gain at the input/output of the dimensionless controller is straightforward, but at the cost of the overall reduction of the uncertain parameters. This is illustrated using the following example.

For instance, in the ACC benchmark example considered above, the controller has displacement as an input and force as an output as shown in Fig. 7 below. Referring to the scaling factors of the two signals, the dimensions obviously do not cancel, i.e. they are related by the stiffness which is an uncertain parameter. In this case, we introduce an additional constant stiffness parameter instead and then the original stiffness parameter will me moved to $B_D$ and will have one more dimensionless uncertain parameter. The newly introduced constant stiffness variable will replace the input-output transformation in Fig. 7 and the original stiffness parameter will now re-cast into the plant. The new plant will have the uncertainty description of the form given by Eq. (10) contrary to Eq. (8). In this case, the method reduces the parametric uncertainty by one instead of by two. Note that $\delta_{\pi 7}$ is due to the uncertain stiffness parameter that is re-cast back to the plant. The study of these limitations is still a work in progress.

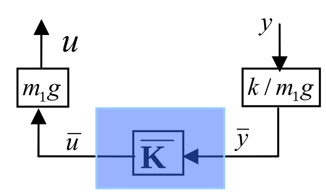


**Fig. 7:** Input-output scaling of the controller

$$\begin{bmatrix} \bar{w} \\ \bar{w}_{\pi 6} \\ \bar{w}_{\pi 7} \end{bmatrix} = \begin{bmatrix} \delta_{\bar{u}} & \delta_{\bar{y}} & 0 & 0 \\ 0 & 0 & \delta_{\pi 6} & 0 \\ 0 & 0 & 0 & \delta_{\pi 7} \end{bmatrix} \begin{bmatrix} \bar{u} \\ \bar{z} \\ \bar{z}_{\pi 6} \\ \bar{z}_{\pi 7} \end{bmatrix} \qquad (10)$$

## SUMMARY

A method called dimensional transformation was presented that for some problems can be used to reduce the number of parametric uncertainty. The method was demonstrated on the 1990 benchmark problem.

It was shown in the general structured uncertainty case that a lower $H_\infty$ norm is achieved by using dimensional transformation to reduce the size of the uncertainty block in the system description. This lower norm should increase the allowable perturbation of the system in the new representation. Also, a better performance in the form of reduced overshoot and control effort was achieved using the dimensionless representation compared to the dimensional representation.

Finally some practical limitations of the method, which are currently under study, were presented. These limitations are highly problem dependent, but are generally mitigated by re-inclusion of uncertainty into the system representation to account for uncertain scaling factors.

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