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### THE VEHICLE AUTOPILOT: SIMULATANEOUS ROBUST CONTROL THROUGH PARAMETRIC ADAPTATION

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### ABSTRACT

This work considers the problem of robustly controlling systems that have an implicit parametric coupling, and specifically considers the problem of lateral control of passenger vehicles at highway speeds. Passenger vehicles collectively have a wide range in dynamic behaviors mainly due to the ranges in size between different models. However, as vehicle size increases, the length, mass and mass moments of inertia also increase in predictable relationships that strongly couple these parameters to each other. The proposed control technique exploits this inherent parametric coupling in order to design a single robust controller that can be easily adapted parametrically from vehicle to vehicle. Parameter decoupling in the design model is achieved in the control synthesis step using a dimensional transformation. The resulting design model presents a system representation suitable for robust control of a very wide range of passenger vehicles using only a dimensional rescaling. This method is distinguished from prior work in that the structure of parametric dependence is included in the controller synthesis. The resulting design is tested on a scaled vehicle test setup developed at Pennsylvania State University. Both simulation and experimental results have shown the effectiveness of the technique for the proposed application.

### **1. INTRODUCTION**

This work discusses a robust, simultaneous control technique for systems whose system parameters are inherently coupled. Human- or naturally-optimized systems will likely exhibit a property where many of the system parameters entering the dynamic model are strongly interrelated. This arises because the key dynamic parameters of a system are generally the same parameters that must be optimized to satisfy design criteria in the system build. A physical example of a collection of systems whose behavior is similar yet scaled along key dynamic parameters is the family of passenger vehicles. For example: a passenger vehicle larger than average tends to

be longer, heavier, and with a larger mass moment of inertia than average as well. Additional generalizations can be made between vehicle size and the tire force generation performance, the suspension behavior, etc. These relationships between length, mass, inertia, etc. obviously do not follow an *exact* functional relationship. But if one simply knows that the system under consideration is a modern production passenger vehicle, one can infer general estimates of many parameters if given just one parameter, mass for instance. This inference can be formalized as equations describing coupling parameter relationships.

The application of a generalized robust control and/or guidance technique in automotive applications is not as extensive as in the aerospace industry, at least as reported in public literature. However, robust control implementation are gaining increased interest in applications of Automated Highway Systems (AHS) [1, 2]. A robust  $H_{m}$  loop-shaping controller was designed in [1] and a nonlinear robust controller was developed for lateral control of heavy trucks in automated highways in [2]. In most vehicle models, the vehicle velocity appears as a free parameter due to the significant changes in the vehicle dynamic model as a function of velocity, changes that sometimes change an open-loop stable model to an unstable model with increasing speeds. Thus, gain-scheduling is often required and used. To address this velocity dependence, a gainscheduling controller was designed in [3] and an LPV controller in [4]. Additional application are described in [5-9].

While scaling theory is an old subject and has been applied to dynamical and structural systems analysis, its application to control of these same systems is very limited and has been seen in literature only during the last decade. One of the most recent and well developed work in this area is the works of Brennan and Alleyne [7, 10, 11]. Previous work by Brennan [7] have shown the advantages of using the dimensionless representation in vehicles for robust control design. Specially, Brennan [7] has shown the achievement of tight frequency-domain variations using dimensionless vehicle models. The tight frequency domain distribution allows for small plant variations from the nominal model finally resulting in smaller uncertainty bounds.

The current work is very different from the previous works in two ways. First, the previous works used a general stackedsensitivity approach to obtain a dynamic uncertainty model. The current work departs from this by modeling system-tosystem variations as a parametric uncertainty. The goal is to obtain a less conservative controller because parametric uncertainty is a subset of the uncertainty seen in the stackedsensitivity approach. Secondly, the current work uses the general  $H_{\infty}$ -synthesis and the  $\mu$ -synthesis/ analysis to better account for structure in the uncertainty model, which was not done in the previous work.

This concept of parametrically constrained engineering systems can be best explained with the help of Fig. 1. Consider three systems that are parametrically different and as indicated in the  $p_1 - p_2 - p_3$  space. These systems are represented by  $G_1, G_2, G_3$  and all enclosed inside the volume S, Fig. 1(a).



Fig. 1: Schematic of systems in parametric space

In the case of passenger vehicles,  $G_1$  may represent a compact car,  $G_2$  mid-size and  $G_3$  a luxury size sedan. Wile one can attempt to design a single robust controller to simultaneously stabilize the three plants, it is often difficult due to the wide range in parameter variation. Additionally, as the number of plants increases, it usually also extends the solid S further such that it may be very difficult if not impossible to synthesize a single controller to stabilize all the plants. Further, to robustly control all systems, one has to encompass all system within an uncertainty description. By bounding parameter variations without considering their coupling, for instance bounding plants  $G_1, G_2, G_3$  with a surface S in the  $p_1 - p_2 - p_3$  space, this inherently includes other plants represented by parametric variation within the same a sphere that encloses S. Many of these parameter combinations can never physically occur, and therefore controllers that consider these plants as key constraints on system performance or robustness may be highly conservative when implemented on the actual systems. While grossly over-simplified, this example illustrates the difficulty in finding a controller that satisfies all robustness and performance conditions for all systems when collective aggregates of dissimilarly sized systems are considered.

The remainder of the paper is organized as follows: First the general framework of the technique is discussed. Next, the

bicycle model is presented for the vehicle dynamic model. Following this, the dimensional transformation method is described. Next, the robust control synthesis and implementation are discussed. Finally, a summary of the main points and results are given.

### 2. FRAMEWORK OF THE TECHNIQUE

The general setup of the current approach is as shown as a general approach in Fig. 2. It can be summarized into four steps:

Step 1 - System transformation: Transform each dimensional model to a dimensionless model using the dimensional transformation operator  $\mathfrak{T}_D$ . One should be judicious in selecting dimensional scaling parameters such that strongly coupled parameters, mass and mass-moment of inertia, appear together as a pi-term in the newly parameterized model. Details on dimensional scaling can be found in most undergraduate fluid dynamics texts.



Fig. 2: The general setup

Step 2 - Perform robust control synthesis: Define a nominal model and uncertainty bound that includes all systems of interest. Also determine the stability and performance requirements on the system in the dimensionless domain through appropriate scaling. Perform the robust control synthesis.

Step 3 - Control system transformation: Transform the dimensionless controller to its corresponding dimensioned controller using the inverse dimensional transformation operator,  $\mathfrak{T}_{p}^{-1}$ .

Step - 4 Verify requirements: Verify that the controller requirements are all met. If requirement not met go back to step 2 and repeat the control synthesis with a different design weights.

### **3. THE BICYCLE MODEL**

Apart from actuator dynamics, the two primary states describing planar vehicle dynamics at constant forward velocity are yaw and lateral motions. A two degree-of-freedom (DOF) planar vehicle dynamic model commonly used is called the bicycle model [7, 12]. In this model, the coupling between the

roll and lateral modes is not considered. The dynamic model is herein expressed in road-fixed error coordinates, Fig. 3.



**Fig. 3:** Schematics of the bicycle model. X-Y and x-y are road (path) and body fixed coordinates, respectively.

While the general bicycle model may have more inputs, only front steering input is considered here. The equation of motion (EOM) of this 2-DOF model is given by Eq. (1).

$$m \cdot \ddot{y} = -\frac{C_{cf} + C_{cr}}{U} \cdot \dot{y} + (C_{cf} + C_{cr}) \cdot \psi$$
$$-\frac{a \cdot C_{cf} - b \cdot C_{cr}}{U} \cdot \dot{\psi} + C_{cf} \cdot \delta_{f} \qquad (1)$$
$$I_{z} \cdot \ddot{\psi} = -\frac{a \cdot C_{cf} - b \cdot C_{cr}}{U} \cdot \dot{y} + (a \cdot C_{cf} - b \cdot C_{cr}) \cdot \psi$$
$$-\frac{a^{2} \cdot C_{cf} + b^{2} \cdot C_{cr}}{U} \cdot \dot{\psi} + a \cdot C_{cf} \cdot \delta_{f}$$

where the state variables are defined as: y: lateral position,  $\dot{y}$ : lateral velocity,  $\psi$ : yaw angle,  $\dot{\psi}$ : yaw rate. The input is  $\delta_f$ : front steering input. The parameters are m: vehicle mass,  $I_z$ : vehicle moment of inertia, U: vehicle longitudinal velocity, a: distance between the center of gravity (C.G.) and the front axle, b: distance between the center of gravity (C.G.) and the rear axle, L: vehicle length between the front and rear axels (=a + b),  $C_{cq}$ : cornering stiffness of the front 2 tires, and  $C_{car}$ : cornering stiffness of the rear 2 tires.

The bicycle model can be represented in state-space form [7], Eq. (2), by choosing the state vector,  $\mathbf{x} \equiv \begin{bmatrix} y & \dot{y} & \psi & \dot{\psi} \end{bmatrix}^T$  and the control input,  $u \equiv \delta_f$ .

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu \tag{2}$$
$$\mathbf{y} = C\mathbf{x} + Du$$

where the system matrices are given by Eq. (3):

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{f_1}{mU} & \frac{f_1}{m} & -\frac{f_2}{mU} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{f_2}{I_2U} & \frac{f_2}{I_2} & -\frac{f_3}{I_2U} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{C_{\alpha f}}{m} \\ 0 \\ \frac{a \cdot C_{\alpha f}}{I_2} \end{bmatrix}$$
(3)
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

with,  $f_1$ ,  $f_2$  and  $f_3$  as defined in Eq. (4)

$$f_{1} = C_{af} + C_{ar}$$

$$f_{2} = a \cdot C_{af} - b \cdot C_{ar}$$

$$f_{3} = a^{2} \cdot C_{af} + b^{2} \cdot C_{ar}$$
(4)

In this system, a, b, L, m, and  $I_z$  tend to increase with increasing size of the vehicle. Therefore, these parameters exhibit a general coupling relationship. Additionally, the front and rear cornering stiffness increase/decrease with mass. Further, front and rear cornering stiffness values are further coupled as they generally change in unison due to uniform road and similar tire conditions.

### 4. THE DIMENSIONAL TRANSFORMATION

The method of dimensional transformation is briefly discussed here. First some preliminaries are presented followed by the transformation of variables, vectors and systems. A detailed procedure can be found in [13].

### 4.1 Preliminaries

The dimensional extraction operator,  $d_{v,e} = D(e,v)$ : is an operator that extracts the units of the variable v relative to the unit system e, a column vector by convention and results in a dimensional unit vector. This dimensional unit vector is nothing but the exponents of each unit used to describe the physical quantity v. To uniquely define this vector, one must specify both the unit system as well as the variable, v. For instance, the gravitational constant  $g = 9.81 \text{ m/s}^2$  has dimensional units that can be represented in many unit systems. For the unit system,  $e = [length mass time]^T$  the extraction operator yields  $d_{v,e} = D(e,g) = \begin{bmatrix} 1 & 0 & -2 \end{bmatrix}^T$ , and in another unit system  $e = [mass force]^T$ , the result of the operator is  $d_{v,e} = D(e,g) = \begin{bmatrix} -1 & 1 \end{bmatrix}^T$ .

The Scaling Matrix,  $A_D$ : is formed by the dimensional unit vectors of the variables (parameters) that are chosen as scaling parameters, also traditionally called repeating parameters.  $A_D$ is a square matrix with rows (and column) size equal to the row size of e. Scaling parameters must be chosen such that  $A_D$  is full rank. For example, for the bicycle model discussed before, choosing the unit system  $e = [length \ mass \ time]^T$  and the scaling parameters  $\{m, L, U\}$ , the scaling matrix  $A_D$  is given by Eq. (5), where the unit system e remains the same for all the parameters and the operator is acted upon each parameter under the same unit system.

$$A_{D} = \begin{bmatrix} d_{M,e} & d_{L,e} & d_{U,e} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$
(5)

The matrix  $A_D$  is defined as a scaling matrix only if the dimensional unit vectors of all the scaling parameters (variables) are extracted with respect to a single unit system e.

### 4.2 Dimensional Transformation of a Variable

Given a variable v, unit systems e and the scaling variables  $[w_1 \ w_2 \ \cdots \ w_n]$ , the dimensional transformation of v to its corresponding dimensionless quantity is defined in Eq. (6).

$$\overline{v} = \Gamma_{\mathbf{D}}(v, e, w) = v \cdot \prod_{i=1}^{n} w_i^{r_v(i)}, \text{ with, } r_v = -d_{v,e}^T \cdot A_D^{-T}$$
(6)

For example, the mass moment of inertia of the bicycle model is transformed to its dimensionless form as follows:

$$r_{I_z} = -d_{I_z,e}^T \cdot A_D^{-T} = -\begin{bmatrix} 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 0 \end{bmatrix}$$

hence,

$$\pi_{5} = \bar{I}_{z} = \Gamma_{\mathbf{D}}(I_{z}, e, w) = I_{z} \cdot \prod_{i=1}^{3} w_{i}^{r_{Cef}(i)}$$
$$= I_{z} \cdot \left(m^{r_{I_{z}}(1)} \cdot L^{r_{I_{z}}(2)} \cdot U^{r_{I_{z}}(3)}\right)$$
$$= I_{z} \cdot \left(m^{-1} \cdot L^{-2} \cdot U^{0}\right) = I_{z} \cdot \frac{1}{mL^{2}}$$

The expressions of all the transformed variables of the bicycle model are summarized in Tab. 1. Note that in these examples  $w = [m \ L \ U]$  and  $e = [length \ mass \ time]^T$ .

Parameters	Signals
$\pi_1 = \frac{1}{L} \cdot a$ $\pi_2 = \frac{1}{L} \cdot b$	$\pi_{6} = \delta_{f} = \overline{\delta}_{f}$ $\pi_{7} = \psi = \overline{\psi}$ $\pi_{8} = \frac{1}{L} \cdot y = \overline{y}$
$\pi_3 = \frac{L}{m \cdot U^2} \cdot C_{\alpha f}$	Time
$\pi_4 = \frac{1}{m \cdot U^2} \cdot C_{\alpha r}$ $\pi_5 = \frac{1}{m \cdot L^2} \cdot I_z$	$\pi_9 = \frac{U}{L} \cdot t = \tau$

 
 Tab. 1: Summary of dimensionless parameters and signals for the bicycle model.

In addition to parameter and signal scaling, all time derivates should take in account the time scaling in the transformation. This can be summarized as follows: given a variable v, a unit systems e and the scaling variables  $[w_1 \ w_2 \ \cdots \ w_n]$ , the dimensional transformation of q repeated derivatives of v with respect to time, i.e.  $\frac{d^q}{dt^q}(v)$ , to

its corresponding dimensionless quantity  $\frac{d^q}{d\tau^q}(\overline{\nu})$  with new time scaling  $(\tau = \beta \cdot t)$ , is defined by Eq. (7).

$$\frac{d^{q}}{d\tau^{q}}(\bar{v}) = \Gamma_{\mathbf{b}}\left(\frac{d^{q}}{dt^{q}}(v), e, w\right) = \beta^{-q} \cdot \frac{d^{q}}{dt^{q}}(v) \cdot \prod_{i=1}^{n} w_{i}^{r_{v}(i)} \qquad (7)$$

$$= \beta^{-q} \cdot \frac{d^{q}}{dt^{q}} \left(\Gamma_{\mathbf{b}}(v, e, w)\right)$$

For example, consider the lateral speed  $(\dot{y})$  of the bicycle model. In this case, q = 1 and  $\tau = \frac{U}{L} \cdot t \implies \beta = \frac{U}{L}$ . And since

$$\overline{y} = \Gamma_{\mathbf{D}}(y, e, w) = \frac{1}{L} \cdot y, \text{ the derivative can be transformed as:}$$
$$\frac{d}{d\tau}(\overline{y}) = \overline{y}' = \Gamma_{\mathbf{D}}\left(\frac{d}{dt}(y), e, w\right) = \beta^{-1} \cdot \frac{d}{dt}(\Gamma_{\mathbf{D}}(v, e, w))$$
$$= \frac{L}{U} \cdot \frac{d}{dt}\left(\frac{1}{L} \cdot y\right) = \frac{1}{U} \cdot \dot{y}$$

## 4.3 Dimensional Transformation of State, Output and Input Vectors

The dimensional transformation of a vector is determined by applying the transformation of component variables to each element of the vector. The focus of this paper is on the three vectors, namely: state, output and input vectors. This transformation can be compactly represented by Eq. (8).

$$\overline{\mathbf{x}} = \mathbf{M}_{\mathbf{x}} \mathbf{x}$$

$$\overline{\mathbf{y}} = \mathbf{M}_{\mathbf{y}} \mathbf{y}$$

$$\overline{\mathbf{u}} = \mathbf{M}_{\mathbf{u}} \mathbf{u}$$
(8)

where,  $M_x$ ,  $M_y$  and  $M_u$  are the state, output and input transformation matrices. For example for the bicycle model, these transformation matrices are given by Eq. (9).

$$\mathbf{M}_{\mathbf{x}} = \begin{bmatrix} \frac{1}{L} & 0 & 0 & 0 \\ 0 & \frac{1}{U} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{L}{U} \end{bmatrix} \qquad \qquad \mathbf{M}_{\mathbf{y}} = \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & 1 \end{bmatrix}$$
(9)  
and 
$$\mathbf{M}_{\mathbf{u}} = \begin{bmatrix} 1 \end{bmatrix}$$

### 4.4 Dimensional Transformation of System Models

The transformation of system models is finally summarized as follows: Given a general plant model  $G \equiv \begin{bmatrix} A & B \\ \hline C & D \end{bmatrix}$  expressed in the dimensional domain with **x**, **y** and **u** as the state vector, output vector and input vector, respectively. The dimensional transformation to an equivalent representation,  $\overline{G} \equiv \begin{bmatrix} \overline{A} & \overline{B} \\ \hline \overline{C} & \overline{D} \end{bmatrix}$ 

expressed in the dimensionless domain with  $\bar{\mathbf{x}}$ ,  $\bar{\mathbf{y}}$ ,  $\bar{\mathbf{u}}$  and  $\beta$  as the new state vector, output vector, input vector and a new time scaling (i.e.,  $\tau = \beta \cdot t$ ), respectively, is as defined as:

$$\overline{G} = \begin{bmatrix} \overline{A} & | \overline{B} \\ \overline{C} & | \overline{D} \end{bmatrix} = \mathfrak{S}_{\mathbf{D}} \Big( G, \mathbf{M}_{\mathbf{x}}, \mathbf{M}_{\mathbf{y}}, \mathbf{M}_{\mathbf{u}}, \beta \Big)$$

$$= \begin{bmatrix} \frac{\beta^{-1} \cdot \mathbf{M}_{\mathbf{x}} A \mathbf{M}_{\mathbf{x}}^{-1}}{\mathbf{M}_{\mathbf{y}} C \mathbf{M}_{\mathbf{x}}^{-1}} & \beta^{-1} \cdot \mathbf{M}_{\mathbf{x}} B \mathbf{M}_{\mathbf{u}}^{-1} \\ \mathbf{M}_{\mathbf{y}} C \mathbf{M}_{\mathbf{x}}^{-1}} & \mathbf{M}_{\mathbf{y}} D \mathbf{M}_{\mathbf{u}}^{-1} \end{bmatrix}$$
(10)

For a control system, the above transformation can be adopted to controller dynamics keeping in mind the input to a controller is an output from the plant and the output from the controller is an input to the plant. More specifically, the transformation is given by Eq. (11).

$$\overline{K} = \begin{bmatrix} \overline{A}_{k} & \overline{B}_{k} \\ \overline{C}_{k} & \overline{D}_{k} \end{bmatrix} = \mathfrak{S}_{\mathbf{D}} \left( K, \mathbf{M}_{\mathbf{kx}}, \mathbf{M}_{\mathbf{y}}, \mathbf{M}_{\mathbf{u}}, \beta \right)$$

$$= \begin{bmatrix} \beta^{-1} \cdot \mathbf{M}_{\mathbf{kx}} A_{k} \mathbf{M}_{\mathbf{kx}}^{-1} & \beta^{-1} \cdot \mathbf{M}_{\mathbf{kx}} B_{k} \mathbf{M}_{\mathbf{y}}^{-1} \\ \mathbf{M}_{\mathbf{u}} C_{k} \mathbf{M}_{\mathbf{kx}}^{-1} & \mathbf{M}_{\mathbf{u}} D_{k} \mathbf{M}_{\mathbf{y}}^{-1} \end{bmatrix}$$
(11)

with  $\mathbf{M}_{xk}$  being the state transformation matrix of the controller state vector and is expressed as:

$$\mathbf{M}_{\mathbf{x}\mathbf{k}} = \begin{bmatrix} \mathbf{M}_{\mathbf{x}} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{\mathbf{x}\mathbf{k}\mathbf{w}} \end{bmatrix}$$
(12)

where,  $\mathbf{M}_{x}$  and  $\mathbf{M}_{xkw}$  are the state transformation matrices of the plant state vector and state vector of the control weights, respectively.

For the bicycle model, the system is transformed using the tools discussed above. The resulting system matrices of the dimensionless representation are given by Eq. (13).

$$\overline{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -p_5 & p_5 & p_6 \\ 0 & 0 & 0 & 1 \\ 0 & p_4 & -p_4 & -p_3 \end{bmatrix}, \quad \overline{B} = \begin{bmatrix} 0 \\ p_2 \\ 0 \\ p_1 \end{bmatrix}$$

$$\overline{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \overline{D} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
(13)

with  $p_1, p_2, \dots, p_6$  as defined in Eq. (14).

$$p_{1} = \frac{\pi_{1}\pi_{3}}{\pi_{5}} \qquad p_{4} = \frac{\pi_{4} - \pi_{1}(\pi_{3} + \pi_{4})}{\pi_{5}}$$

$$p_{2} = \pi_{3} \qquad p_{5} = \pi_{3} + \pi_{4}$$

$$p_{3} = \frac{\pi_{1}^{2}(\pi_{3} + \pi_{4}) + \pi_{4}(1 - 2\pi_{1})}{\pi_{5}} \qquad p_{6} = \pi_{4} - \pi_{1}(\pi_{3} + \pi_{4})$$
(14)

# 5. THE ROBUST CONTROL SYSNTHESIS AND IMPLEMENTATION

In order to perform the robust control synthesis, the dimensionless system needs to be represented in LFT form. This form basically separates the nominal model from the uncertainty. The nominal model is determined from Fig. 4 and is given by Eq. (15).

$$\overline{y}'' = -p_{50}\overline{y}' + p_{50}\psi + p_{60}\psi' + w_2 - w_5 + w_6 + p_{20}\delta_f$$

$$\overline{\psi}'' = p_{40}\overline{y}' - p_{40}\psi - p_{30}\psi' + w_1 - w_3 + w_4 + p_{10}\delta_f$$
(15)



Fig. 4: Block diagram representation of the LFT dimensionless bicycle model

The nominal values of the parameters  $p_1 - p_6$  in this model are obtained by using means of the pi-parameters calculated for many different passenger vehicles [7, 14].

The uncertainty block that relates the disturbances  $\overline{\mathbf{z}} = \begin{bmatrix} z_1 & \dots & z_6 \end{bmatrix}^T$  and  $\overline{\mathbf{w}} = \begin{bmatrix} w_1 & \dots & w_6 \end{bmatrix}^T$  is a diagonal and given by Eq. (16).

$$\overline{\Delta} = diag\left(\begin{bmatrix} \delta_{p1} & \delta_{p2} & \delta_{p3} & \delta_{p4} & \delta_{p5} & \delta_{p6} \end{bmatrix}\right)$$
(16)

### 5.1 The H<sub>a</sub> Synthesis

The controller synthesis is performed using the  $H_{\infty}$  control synthesis. The design criteria are as follows:

• Robust stability for all  $\pi$  variations  $|\delta_{\pi i}| \le 0.2$ . The bound on  $\pi$  parameters is chosen such that it covers variations in parameters of many passenger vehicles as shown by statistical distribution in [7, 14].

Robust performance:

- $\circ$  For an impulse lateral force, the unitless lateral displacement should be less than 0.15m/m and a unitless settling time less than 8 sec/sec. Also the control action should have its magnitude less than 0.2 rad (~ 11.5°),
- $\circ$  For an impulse yaw moment, the yaw angular displacement should be less than 0.2 (~ 11.5°) and a unitless settling time less than 8 sec/sec. Also the control action should have its magnitude less than 0.2 rad (~ 11.5°).

The simulated response to an impulsive lateral force and yaw moment are given by Fig. 5 and Fig. 6 respectively. In both cases the above requirements are met.



**Fig. 5:** Response to an impulsive lateral force: (top) lateral position, (bottom) control action.



Fig. 6: Response to an impulsive moment: (top) yaw angle, (bottom) control action.

### 5.2 The $\mu$ -Synthesis/Analysis

The uncertainty block in this problem clearly shows that it has a diagonal structure and the  $H_{\infty}$  controller designed in the previous section is expected to be conservative as it doesn't account for the structure of the actual uncertainty. Therefore, another synthesis approached that takes into consideration the structure of the uncertainty, known as the  $\mu$ -synthesis may be necessary depending on the results of the  $\mu$ -analysis. A  $\mu$ -analysis performed on the above  $H_{\infty}$  controller to decide if  $\mu$ -synthesis is necessary. The results show that it's not required as the structured and unstructured singular values have close values. A summary of the analysis result is given in Tab. 2.

	Robust stability	Robust stability/Nominal performance
$H_{\infty}$ Norm	0.671	1.012
$\mu$ -upper bound	0.628	0.748
$\mu$ -lower bound	0.627	0.705

 Tab. 2: Summary dimensionless parameters and signals of the bicycle model

#### 5.3 Experimental implementation

The proposed technique is tested using a  $1/5^{\text{th}}$  scaled vehicle on a rolling roadway simulator shown in Fig. 7. The vehicle has been designed such that dimensionless tire and inertial properties at the speed of operation match those of a full sizevehicle at 15 m/s. The controller is implemented using SIMULINK and compiled using real time workshop with hardware target. The dimensionless controller designed for a generalized vehicle is transformed to dimensioned form by applying the inverse of the transformations of Eq. (6) and (7) using the mass, length, and velocity scaling factors specific to the scale vehicle (Step 3 of Fig. 2).



Fig. 7: The scaled vehicle and rolling roadway simulator.

The vehicle is equipped with a steering actuator and position sensors. The position and orientation of the vehicle is measure indirectly using a linkage with encoders located at each joint. The position and orientation are calculated from the encoder readings and the dimensions of the linkage.

Two scenarios were considered in both the numerical and experimental cases: 1) driving with smooth lane change and 2) driving under sudden lane change and maneuvering. The first operation resembles the normal driving condition on highways and city where lane-change is done merely to overtake slow vehicles on a lane. The performance of the controller on the nominal plants is considered effective as shown in Fig. 8. In the second scenario, the controller performance under sudden change of lane is evaluated. Sudden change of lane usually occurs in avoiding accidents that happen instantly in front of the vehicle or sudden change of mission, in the case of high performance vehicles: such as military vehicles, that required a maneuvering to suddenly change in position. This scenario is an extreme case and is the most critical to evaluate the performance of the robust controller. Fig. 9 show the response to the driving condition under sudden change of lane.



Fig. 8: Response to driving under smooth lane changing.



Fig. 9: Response to driving under sudden lane changing.

The performance of the controller in both scenarios is compared to the experimental results of the scaled vehicle. Note that the controller is universal in the sense that it was NOT designed for the scale vehicle. Instead, it is designed for many vehicles within the family of passenger vehicles in the dimensionless domain, and the resulting dynamic controller is re-scaled as needed to various vehicles of different sizes. Therefore, if one wants to test it on another vehicle in the family, say a bigger vehicle, one has to transform only the controller based on the parameters of the new vehicle. To use wording that parallels "gain-scheduling", this method is "plantscheduling" a robust controller from one size vehicle to another; the universal controller is parametrically adapted to every vehicle in the family.

### 6. SUMMARY

The paper focused on the development of a technique for robust control and experimental implementation using the design of a robust vehicle autopilot as an example. The design and dimensional transformation process was discussed and the effectiveness of the method as an alternative approach to conventional simultaneous stabilization control. The effectiveness of the controller was demonstrated both numerically and experimentally. By use of dimensional scaling, the controller synthesis accounts for much of the general coupling between the parameters. Significant parameter variation from systems of widely different sizes is accounted for by transforming the controller back to the dimensioned domain through dimensional transformations specific to each system.

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