

# Investigation of the Effect of Continuously Variable Transmissions on Ground Robot Powertrain Efficiency

Jesse Pentzer and Sean Brennan

**Abstract**—Explosive ordnance disposal (EOD) robots are limited in endurance and range by the amount of energy available in the batteries used to power them. Continuously variable transmission (CVT) technology has developed quickly in recent years in the automotive field and is now being applied to smaller vehicles such as bicycles and electric scooters. This paper will discuss simulations investigating the feasibility of adding CVT transmissions to robot powertrains in order to improve the overall efficiency of the drive system. The equations used to calculate the power required to move a robot at varying speeds will be described, as well as the equations used to model CVT and direct-drive transmissions, DC motors, and power discharge from a battery. The results of a constant mass simulation, where the added CVT mass was offset by a loss in battery mass, showed that adding CVTs is not a feasible option due to the mass of the CVTs. In an added mass scenario, where the mass of the CVTs was added to the overall robot mass, the benefits of a CVT depended strongly on the speeds at which the robot was expected to perform. A robot expected to operate at low speeds most of the time would benefit more from a CVT than a robot expected to operate near maximum speed most of the time.

## I. INTRODUCTION

As of 2007, 70% of all American combat casualties in Iraq and 50% of combat casualties in Afghanistan were caused by suicide car bombs, roadside bombs, and improvised explosive devices (IEDs) [1]. Explosive ordnance disposal (EOD) robots play an important role in efforts to combat these threats, but the endurance of these robots is limited by the batteries used to power them [1]. Improving battery storage capabilities and carrying extra batteries helps to mitigate this problem, but soldiers are already carrying as much as 16 pounds of various battery types to power other equipment [2]. A third method of extending robot endurance is improving the efficiency of the robot powertrain itself. Along with investigating alternate energy sources, developing more efficient equipment is an important focus of the armed forces [3].

Robot powertrains commonly consist of a battery, direct current (DC) electric motors, a gear reduction system, and driving wheels or tracks. Gear reduction is usually achieved using chains and sprockets, geared transmissions, or a combination of the two. These methods of gear reduction provide a fixed gear ratio, so the speed of the vehicle is changed by adjusting the rotational speed of the motor. Because DC

motor efficiency is directly related to motor speed, the motor will only operate near peak efficiency over a small range of vehicle speeds.

The gear ratio of a continuously variable transmission (CVT) is infinitely adjustable between minimum and maximum values. Once installed in a robot powertrain, this adjustable gear ratio allows the motor to operate at a constant speed while the speed of the transmission output is changed. Thus, the motor can be operated at a speed near peak efficiency over a larger range of transmission output speeds. While research in CVT efficiencies and the implementation of CVTs in electric vehicle powertrains has been reported in previous literature, there is little research applying this technology to ground robots. A comparison of CVT technologies concluded that rubber belt CVTs are generally the most efficient design [4]. Belt CVTs suffer from torque losses due to radial belt slippage and speed losses due to belt creep and shear deflection, none of which affect geared transmissions [5]. It is interesting to note that the operating conditions of high input speed with low input torque, conditions at which DC motors are most efficient, represented the lowest regions of CVT efficiency [4]. Specifically related to ground robots, previous work has analyzed the optimal control and efficiency of a spherical-CVT equipped robot [6]. The research presented here is different in that load models created using experimental EOD robot data are utilized and the CVT modeled is a commercially available design that provides a range of gear ratios different than the range of the spherical-CVT.

The purpose of this study is to investigate the efficiency improvements gained by robots using a CVT versus a direct-drive, or fixed gear ratio, DC motor powertrain. The following two sections describe the methods used to simulate the robot powertrain. Simulation results are discussed in Section IV, and Section V presents the efficiencies calculated for the robot based on field measurements. Finally, Section VI discusses conclusions drawn from this work.

## II. POWERTRAIN SIMULATION

This section describes the equations used to model a robot powertrain. The analysis is divided into four sections: one to determine power loss from ground and track resistance, one to model the CVT and fixed gear ratio transmissions, one to model a DC electric motor, and one to model battery discharge.

J. Pentzer is with the Department of Mechanical and Nuclear Engineering, The Pennsylvania State University, State College, PA, 16802 jlp5573@psu.edu

S. Brennan is with the faculty of mechanical and nuclear engineering, Department of Mechanical and Nuclear Engineering, The Pennsylvania State University, State College, PA, 16802 sbrennan@psu.edu

### A. Ground and Track Resistance

To model the power required to move a robot at a given speed, equations were derived from experimental power discharge data collected from tracked and wheeled robots operating on varying terrain types [7]. For a tracked robot, the equation was

$$P_{loss,g-t} = m_{robot} (k_{terrain} \cdot V_{robot} + b_{terrain}), \quad (1)$$

where  $P_{loss,g-t}$  is the power loss due to ground-track interaction in watts,  $m_{robot}$  is the mass of the robot in kilograms, and  $V_{robot}$  is the speed of the robot in meters per second. The constants  $k_{terrain}$  and  $b_{terrain}$  varied with the type of terrain being traversed. Table I gives the constants used in this research [7].

The power loss from forces in the track resisting turning by the drive sprocket were also modeled from experimental data. The equation used was

$$P_{loss,r-t} = 0.01268 \cdot m_{robot} \cdot V_{robot}^{2.06}, \quad (2)$$

where  $P_{loss,r-t}$  is the power loss in watts due to track internal resistance, and the units of  $m_{robot}$  and  $V_{robot}$  are kilograms and meters per second, respectively.

### B. CVT and Fixed Gear Ratio Transmission Models

A commercially available CVT designed for use on bicycles and light electric vehicles was chosen for this research. The CVT modeled provided gear ratios from 2.0 through 0.57, where the gear ratio,  $GR$ , was defined to be the ratio of the input shaft speed to the output shaft speed:

$$GR = \frac{\omega_{in}}{\omega_{out}}. \quad (3)$$

Vendors generally provide efficiency data for a combination of motor and gearbox. Using the motor model presented previously, the efficiency of the gearbox alone can be calculated using

$$\eta_{gear} = \frac{\eta_{total}}{\eta_{motor}} \quad (4)$$

where  $\eta_{gear}$  is the transmission efficiency,  $\eta_{total}$  is the system efficiency reported by the manufacturer, and  $\eta_{motor}$  is the efficiency of the motor modeled using the techniques in Section II-C. In previous work, vendor data for motor-transmission efficiencies was utilized to calculate transmission efficiencies as a function of gear ratio [7]. The efficiency of the CVT was modeled using the relationship

$$\eta_{CVT} = -0.06 \cdot \ln(GR_{CVT}) + 0.93, \quad (5)$$

TABLE I  
CONSTANTS FOR SOIL-TERRAIN POWER LOSS EQUATION.

Terrain	$k_{terrain}$ (m/s <sup>2</sup> )	$b_{terrain}$ (m <sup>2</sup> /s <sup>3</sup> )
Asphalt	1.18	0.15
Grass	1.21	0.11
Tile	1.00	0.20
Dirt	1.12	0.31
Gravel	0.81	0.01
Brush	1.53	0.66

where the efficiency of the CVT was  $\eta_{CVT}$  and the gear ratio was  $GR_{CVT}$ . When the CVT gear ratio dropped below 1.0, the inverse of the gear ratio was used for  $GR_{CVT}$  in (5). This is because it was assumed that a transmission would have the same efficiency when increasing or decreasing the output speed by the same gear ratio. As an example, a 20:1 transmission, meaning 20 input revolutions to one output revolution, would have the same efficiency as a 1:20 transmission, even though the gear ratio of the first is 20 and the gear ratio of the second is 0.05. A plot of the calculated CVT efficiency over the full range of CVT gear ratios is given in Fig. 1. Reported efficiencies for other friction-drive CVTs indicate that the efficiency calculated from (5) will be higher than the actual efficiency of the transmission [8], [5]. Therefore, in the absence of efficiency data on the CVT being modeled, (5) provides a best-case estimate of the CVT efficiency at a given gear ratio.

A fixed gear ratio transmission was needed with the CVT to allow the motor to run most efficiently. Generally a fixed gear ratio of approximately 20-30 was required for the motor to run near peak efficiency. Hence, the CVT gear ratio range of 2.0 to 0.57 was not adequate alone. The fixed gear ratio transmission efficiency was modeled using (5) with  $\eta_F$  and  $GR_F$  in place of  $\eta_{CVT}$  and  $GR_{CVT}$ , respectively.

### C. DC Electric Motor Model

The DC electric motor was modeled as running at steady state given the necessary output power,  $P_{m,out}$ , and angular rate,  $\omega_m$ . The motor output power was calculated from

$$P_{m,out} = \frac{P_{move}}{\eta_{CVT} + \eta_F} \quad (6)$$

where  $P_{m,out}$  was calculated using

$$P_{m,out} = P_{loss,g-t} + P_{loss,r-t}. \quad (7)$$

The required motor angular rate,  $\omega_m$ , was calculated using

$$\omega_m = \frac{V_{robot}}{r_w \cdot GR_{CVT} \cdot GR_F} \quad (8)$$

where  $r_w$  is the radius of the drive wheel of the robot.

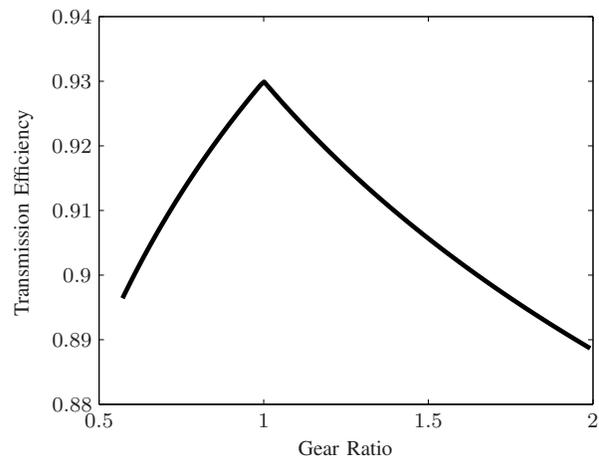


Fig. 1. Modeled CVT efficiency over full range of gear ratios.

The necessary voltage,  $V$ , to drive the motor at the required speed was calculated using

$$V = V_{emf} + \frac{R_a}{V_{emf}} \left( P_{m,out} + \frac{V_{emf}^2}{R_h} \right) \quad (9)$$

where  $R_a$  and  $R_h$  are constant properties of the motor. The motion of the spinning motors, like a generator, causes a voltage opposed to the applied voltage. This is called the back-EMF,  $V_{emf}$  [9]. The  $R_a$  would normally be supplied by motor manufacturers, and  $R_h$  was calculated using

$$R_h = \frac{K \cdot \omega_0}{I_0} \quad (10)$$

where  $K$  is the motor constant,  $\omega_0$  is the no-load motor speed, and  $I_0$  is the no-load motor current. The back-EMF,  $V_{emf}$ , was calculated using

$$V_{emf} = \omega_m \cdot K. \quad (11)$$

The power into the motor,  $P_{m,in}$ , was calculated using

$$P_{m,in} = \frac{V(V - V_{emf})}{R_a}. \quad (12)$$

Using the output and input motor power, the efficiency of the DC motor was calculated from

$$\eta_m = \frac{P_{m,out}}{P_{m,in}}. \quad (13)$$

The motor constants  $K$ ,  $\omega_0$ ,  $I_0$ , and  $R_a$  and the optimal fixed gear ratio,  $GR_F$ , were calculated using relations described in Section III.

#### D. Battery Discharge Model

The power drawn from the batteries on the robot is assumed to be the same as the power input to the motor calculated in (12). The change in the state of charge of a battery is dependent on how fast the battery is being discharged [10]. Also, the change in state of charge is a nonlinear function of the energy contained in a fully charged battery. As battery mass is adjusted during simulation, it is necessary to model the change in energy based on battery capacity and not as a function of battery mass alone assuming proportionality. The capacity of a battery, generally given in units of amp-hours by the manufacturer, is given for a specific discharge time. For example, a battery with a capacity of 10 amp-hours and a rating time of 10 hours will provide 1 amp for 10 hours. If the battery is discharged at 2 amps, it will last less than 5 hours, and if the battery is discharged at 0.5 amps, it will last more than 20 hours. A constant called the Peukert number is used to model battery discharge and varies based on the type of battery being modeled.

The first step in modeling battery discharge was the calculation of a constant,  $k_b$ , using

$$k_b = V_{bat} \cdot \frac{C}{R} \cdot \Delta t_s \cdot R^{\frac{1}{N}} (\Delta t_h \cdot E_{nom})^{-\frac{1}{N}} \quad (14)$$

where  $V_{bat}$  is the battery voltage,  $C$  is the battery capacity in amp-hours,  $R$  is the capacity rating time in hours,  $\Delta t_s$

is the time step over which the discharge occurs in seconds,  $N$  is the Peukert number for the battery type,  $\Delta t_h$  is  $\Delta t_s$  converted into hours, and  $E_{nom}$  is the energy contained in a fully charged battery in joules. Using  $k_b$ , the change in the state of charge of the battery was calculated using

$$\Delta E_{soc} = \left( \frac{E_{sup}}{k_b} \right)^N \quad (15)$$

where  $E_{sup}$  is the energy drawn from the battery during the current time step and was calculated from

$$E_{sup} = P_{m,in} \cdot \Delta t_s. \quad (16)$$

### III. MOTOR AND TRANSMISSION DESIGN EQUATIONS

The DC motor was designed based on the power the motor was required to output at maximum efficiency. This power was calculated using (1) and (2) with a baseline motor mass, assumed to include all components except the motor and fixed gear ratio transmission, and a maximum desired speed,  $V_{robot,m}$ , of 2.5 m/s. Equations (17)-(20) used the required power,  $P_{req}$ , to calculate the motor mass,  $m_{motor}$ , motor constant,  $K$ , no-load current,  $I_0$ , and armature resistance,  $R_a$  [7]. A model of the motor at the no-load condition produced (21), which was used to solve for the no-load motor speed,  $\omega_0$ , and ensure consistency between the motor constants [9].

$$m_{motor} = 0.0386 \cdot P_{req}^{0.7098} \quad (17)$$

$$K = -1 \cdot 10^{-9} \cdot P_{req}^2 + 3 \cdot 10^{-5} \cdot P_{req} + 0.0323 \quad (18)$$

$$I_0 = 5 \cdot 10^{-6} \cdot P_{req}^2 + 0.0028 \cdot P_{req} + 0.1589 \quad (19)$$

$$R_a = 41.245 \cdot P_{req}^{-1.005} \quad (20)$$

$$\omega_0 = \frac{I_0 \cdot R_a - V_{max}}{-K} \quad (21)$$

The value of  $V_{max}$  was the maximum voltage that would be applied to the motor. The speed at which the motor achieved maximum efficiency,  $\omega_{eff,max}$ , was found by incrementing the motor speed from zero to the no-load speed,  $\omega_0$  and calculating the motor efficiency using the equations in Section II-C. The required angular rate of the driving wheels,  $\omega_{drive}$  was then calculated using (22). The ratio of the maximum efficiency motor speed to the required driving speed gave the gear ratio to achieve the relative speeds, as shown in (23).

$$\omega_{drive} = \frac{V_{robot,m}}{r} \quad (22)$$

$$GR_F = \frac{\omega_{eff,max}}{\omega_{drive}} \quad (23)$$

Another relation, developed using vendor data on motor and transmission masses, was used to calculate an approximate transmission mass based on the required gear ratio. Equation (24) gives the gearbox mass fraction,  $GB_{mf}$ , as a function of the gear ratio. This percentage was multiplied by the calculated motor mass to find the gearbox mass,  $m_{GB}$ , as shown in (25).

$$GB_{mf} = 0.004 \cdot GR + 0.560 \quad (24)$$

$$m_{GB} = GB_{mf} \cdot m_{motor} \quad (25)$$

The actual robot mass was calculated by adding the motor and transmission masses to the baseline robot mass, as shown in (26).

$$m_{robot} = m_{motor} + m_{GB} + m_{robot-base} \quad (26)$$

The total robot mass calculated in (26) was not the same as the mass used to calculate  $P_{req}$ , size the motor, or size the transmission. Because of this, all the steps in designing the motor and gearbox were iterated until the calculated values stopped changing. Convergence was usually achieved within five iterations.

#### IV. SIMULATIONS AND RESULTS

The first step in simulating the powertrain was to determine the parameters for the robot model. For this initial research, generic values were assumed and are given in Table II. The relations in Section III were then applied to design the motor and transmission, producing the values in Table III.

The next step was to calculate the efficiency of the system as a function of robot speed. Fig. 2 shows the efficiency of both the direct drive and CVT equipped powertrains. Below speed values of 0.5 m/s the CVT provided significant efficiency improvements, but at speeds above 0.5 m/s the direct drive powertrain had a higher efficiency. The latter was due to the fact that the transmission was designed to allow the motor to operate at maximum efficiency at the cruising speed, 2.5 m/s. The motor in the CVT equipped powertrain also ran at peak efficiency, but the inclusion of the CVT power losses lowered the overall efficiency of the powertrain. Fig. 3 shows the DC motor efficiency as a function of robot speed. The CVT equipped robot ran the motor at peak efficiency over 85% of the speeds and also increased the maximum speed by 20% over the direct drive robot.

The CVT gear ratio was adjusted as shown in Fig. 4 to run the motor at maximum efficiency. At low speeds, the CVT operated with a higher gear ratio, allowing the motor to spin faster than the driving wheels. As robot speed increased, the gear ratio was reduced steadily until reaching a value of one. The plateau at a gear ratio of one occurred because the CVT efficiency was highest at that gear ratio, which is evident in Fig. 1. The CVT gear ratio was below one at speeds greater than 1.6 m/s, resulting in the output shaft spinning faster than the input shaft.

Using the calculated powertrain efficiencies at varying robot speeds, two scenarios for implementing a CVT on a robot were investigated: a constant mass scenario and an added mass scenario.

##### A. Constant Mass

For the constant mass simulation, the total mass of the robot was constrained to remain constant despite the addition of the CVT. To offset the added CVT mass, an equivalent mass of batteries was removed. To analyze this design scenario, the battery discharge of both the direct drive and

TABLE II  
ROBOT PARAMETERS.

Parameter	Value
Baseline Mass, $m_{robot-base}$	100 kg
Drive Wheel Radius, $r_w$	0.1 m
Number of Motors	2
Maximum Speed, $V_{robot,m}$	2.6 m/s
CVT Gear Ratio Range	2 - 0.57
CVT Mass	3.8 kg
Peukert Number, $N$	1.1
Battery Rating Time, $R$	20 hours
Battery Voltage, $V_{bat}$	28.8 V
Battery Capacity, $C$	7.2 amp-hours

TABLE III  
DC MOTOR AND TRANSMISSION PARAMETERS.

Parameter	Value
No-Load Speed, $\omega_0$	714 rad/s
No-Load Current, $I_0$	1.04 amps
Armature Resistance, $R_a$	0.17 ohms
Motor Constant, $K$	0.039 $\frac{N \cdot m}{amp}$
Fixed Gear Ratio, $GR_F$	27

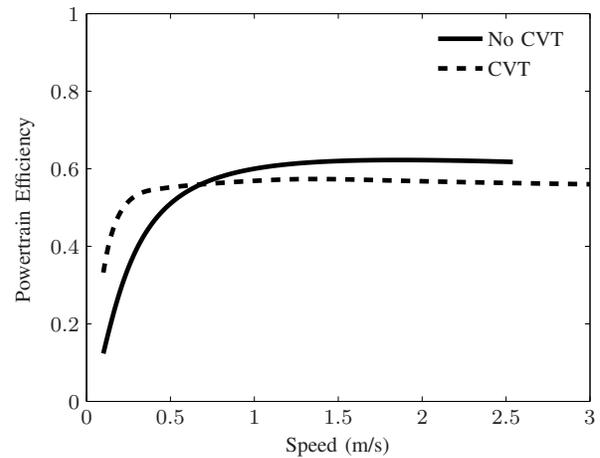


Fig. 2. Powertrain efficiency over increasing robot speeds.

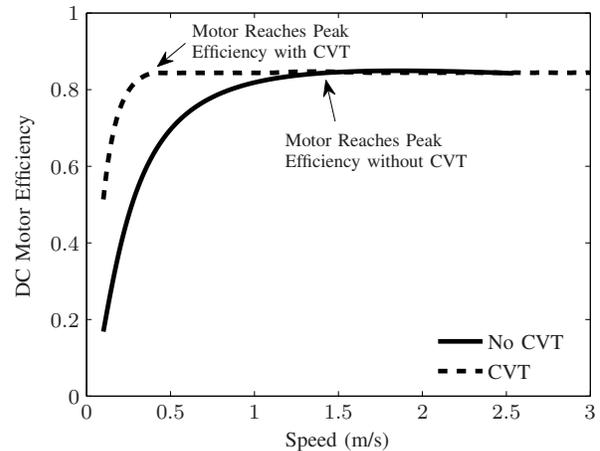


Fig. 3. DC motor efficiency over increasing robot speeds.

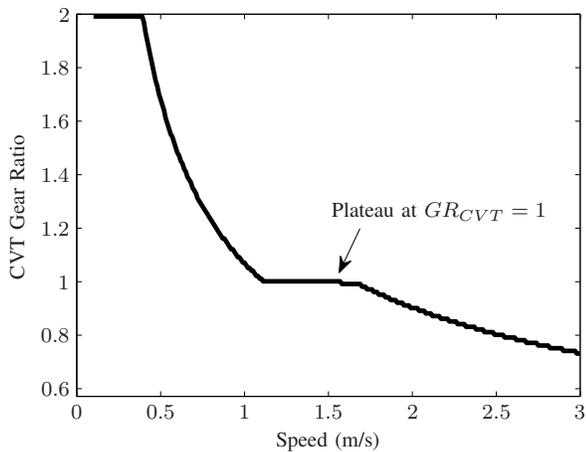


Fig. 4. CVT gear ratio to maintain maximum motor efficiency.

CVT equipped robots was modeled. To have a successful design, the energy saved by using the CVT would need to exceed the energy lost in the replaced battery mass. The battery discharge was modeled with the robot moving at 0.16 m/s. This was done because, as shown in Fig. 2, the CVT equipped robot had the greatest efficiency advantage at that speed. The resulting battery discharge curves are given in Fig. 5. The results show that while the CVT robot was using less power, it was not saving enough energy to make up for the large amount lost in the replaced battery mass. At the point where the CVT robot battery pack was depleted, 36% of the initial energy offset had been recovered. The mass of each CVT, 3.8 kg, is much larger than the mass of a BB2590 battery, 1.4 kg. To remain at constant mass, approximately two batteries must be removed with each added CVT. This represents a significant loss of energy, and becomes impossible on small robots that require much less than 3.8 kg of batteries for operation in the standard configuration.

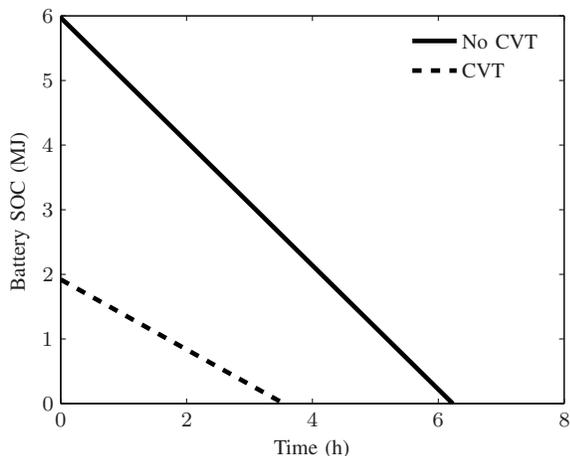


Fig. 5. Battery discharge with robot speed of 0.16 m/s in constant mass analysis.

## B. Added Mass

The second design scenario considered was adding the CVT mass to the robot mass with no compensation through loss of battery mass. In this case, the penalty for adding the CVT mass was an increase in the power required to move the robot at a given speed, as shown in Fig. 6. The powertrain of the robot was designed again using the equations in Section III, taking into consideration the added CVT mass. Using the new powertrain efficiency values, the power used by the motor to propel the robot at varying speeds was calculated. The results, shown in Fig. 7, reflect the fact that the CVT equipped robot was more efficient below 0.5 m/s and less efficient above 0.5 m/s. The added mass scenario appears to be most plausible if the robot was expected to operate at low speeds a majority of the time.

## V. EFFECTIVE POWERTRAIN EFFICIENCY

As discussed previously and shown in Fig. 2, the overall efficiency of the powertrain is a function of robot speed. To determine an effective powertrain efficiency, field measurements of robot speed were recorded while performing

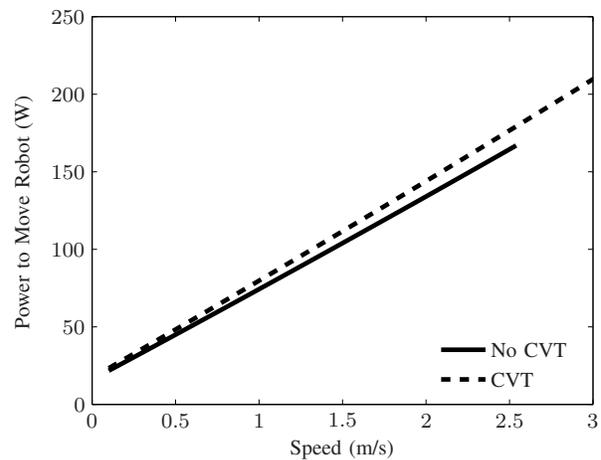


Fig. 6. Power to move robot at increasing speeds in added mass analysis.

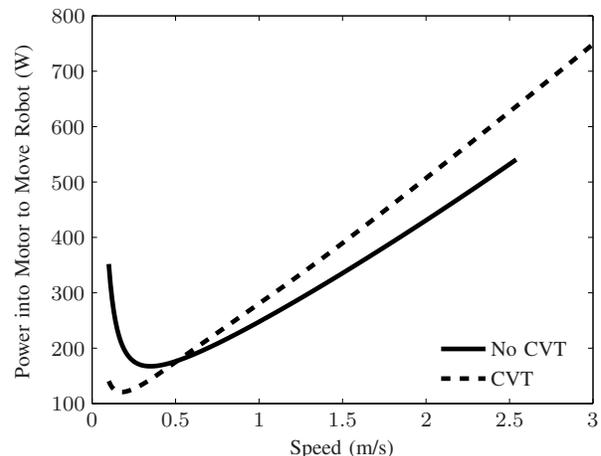


Fig. 7. Power entering motor to move robot at increasing speeds in added mass analysis.

standard mission tasks. These tasks included climbing stairs, traversing hard and soft terrain, climbing obstacles, and inspecting a given area. The velocity data was used to determine the percent of total mission time spent by the robot in increasing speed ranges, as shown in Fig. 8. The measurements indicate that the majority of a mission is spent with the robot moving slowly. The histogram results are used to calculate an effective efficiency,  $\eta_o$ , using

$$\eta_o = \sum_{k=1}^n \eta_k \cdot T_n \quad (27)$$

where  $\eta_k$  is the average efficiency over the  $k^{th}$  velocity bin in the histogram,  $T_n$  is the percent of mission time the robot operated in the  $k^{th}$  velocity bin, and  $n$  is the number of velocity bins in the histogram. The average efficiency,  $\eta_k$ , was calculated from the results in Fig. 2. The powertrain efficiency at zero speed was assumed to be zero because no power is used for moving the robot. For the speed profile given in Fig. 8, the effective efficiency of a CVT equipped robot is 29% while the effective efficiency of a robot without a CVT is 23%. The addition of the CVT improved overall powertrain efficiency by roughly 25% compared to the direct drive design.

The choice of adding a CVT or increasing battery mass to improve robot endurance appears to hinge on the type of mission the robot is expected to perform. A robot operated at one speed the majority of the time could be designed to operate with maximum efficiency at that speed and would benefit most from increasing battery mass. An example of this would be a robot used to carry loads from one place to another repeatedly over a known path. Robots with large accessory loads would also benefit more from increasing battery mass than adding CVTs. As accessory loads are continuously drawn from the batteries, increasing battery mass would most likely improve endurance more than the low-speed efficiency gains of adding CVTs. On the other hand, an EOD robot may benefit more from having a CVT

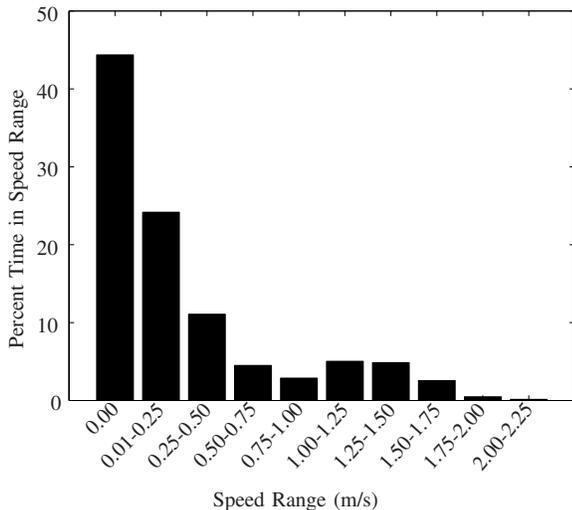


Fig. 8. Histogram obtained from speed measurements showing percent of mission time in a given speed range.

because it is expected to operate over a wide range of speeds. For example, it may be necessary to drive quickly to a location and then slowly inspect the area and buildings for signs of danger. Any robot that operates slowly the majority of the time but must maintain the ability to move quickly would benefit from a CVT.

## VI. CONCLUSIONS

The results of the simulations described in this paper indicate that adding a CVT to a robot powertrain is most feasible when the CVT mass is added to the robot mass and not offset with a loss in battery mass. In this configuration, the CVT provides an increase in the maximum speed of the robot and improves the powertrain efficiency at lower speeds. A significant factor impacting the results of these simulations was modeling the CVT efficiency as a direct drive transmission. These results should be considered as a best case scenario, and in practice the CVT will most likely be less efficient than the one modeled here. With this in mind, a CVT appears to be most beneficial when a robot is expected to operate at low speeds the majority of the time but must still retain the capability to move at higher speeds.

Speed measurements recorded while a robot performed tasks such as climbing stairs or traversing to a specified point indicate that the majority of mission time is spent operating at low speeds. The powertrain efficiency was improved from 23% to 29% with the addition of the CVT. Continued work in this research will involve improving CVT efficiency modeling and additional measurements of robot speed during simulated missions.

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