The Pennsylvania State University The Graduate School College of Engineering

VEHICLE CONTROL FOR COLLISION AVOIDANCE AND ROLLOVER PREVENTION USING THE ZERO-MOMENT POINT

A Thesis in

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by

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Abstract

This thesis investigates vehicle control techniques for rollover prevention in a collision avoidance scenario. The zero-moment point (ZMP) is used to evaluate the vehicle's current and near-future rollover propensity with the purpose of predicting and correcting an impending rollover event. Low-order vehicle models inclusive of roll dynamics and terrain effects are utilized to facilitate rapid, but accurate, calculation of the vehicle's current and predicted rollover threat. Results found in this thesis show that short-range predictions, ranging from 0.1 seconds to 0.7 seconds, are sufficient to prevent nearly all dynamics-induced rollovers in typical highway curves. These results are useful in determining an appropriate preview horizon for predictive control techniques aimed at rollover prevention. This thesis also investigates the minimum intervention distance needed in a collision avoidance scenario to avoid an obstacle while also preventing wheel lift. Subsequently, a linear-quadratic output regulator is designed to safely navigate the vehicle through a collision avoidance maneuver, while employing a feedback scheme that explicitly accounts for rollover prevention.

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Chapter 1 Introduction

1.1 Motivation

Safety has long been considered one of the primary concerns in automobile design due to the inherent danger of vehicle travel. According to the National Highway Traffic Safety Administration (NHTSA), in 2012 there were over 6.5 million reported crashes, resulting in over 33,000 deaths [1]. While the number of fatal crashes has been steadily decreasing over the past decade, vehicle-related accidents remain the leading cause of death for people between the ages of 11 and 27 [1]. In particular with regards to this research, vehicle rollover is a notoriously deadly form of automobile accident, accounting for over 35% of all fatalities [1].

Until recently, almost all safety mechanisms were reactive in nature; that is, designs focused on keeping the occupant safe during a collision (seatbelts, air bags, etc.). Semiand fully autonomous vehicles, however, have the potential to greatly reduce the number of vehicle accidents by responding proactively to a situation, taking action before it is too late. Examples of such technologies already appearing on production vehicles are adaptive cruise control (ACC), lane departure warning (LDW) systems, electronic stability control (ESC) [2], and rear-end collision braking systems [3]. Modern sensors and actuators that aid in the driving task are able to diagnose and respond to an emergency faster than a human is capable. For this reason, the focus of many driver-assist features has been safety-oriented with the hope of continuing to reduce human loss from automobile accidents.

1.2 Research Goals

Several active safety systems in particular have seen growing interest in the intelligent vehicles community: stability control, lane-keeping, collision avoidance, and rollover prevention. Determining an effective trajectory (and subsequent vehicle control along this trajectory) remains one of the greatest challenges in this research field. In an emergency situation, all of these systems must work together seamlessly in order to avoid a tragedy.

Although these safety systems are heavily studied individually, there is limited literature on systems that consider stability control, lane-keeping, collision avoidance, and rollover prevention simultaneously. An effective emergency maneuver not only requires the vehicle to avoid an obstacle while remaining on the road, but to also prevent wheel lift and maintain yaw stability. Therefore, it is the hope of this research to progress the development of emergency vehicle control algorithms by investigating the effects of rollover in a collision avoidance scenario. These effects are particularly important for rollover-prone vehicles such as heavy trucks or SUVs.

Specifically, this research uses a linear-quadratic control framework to maneuver a vehicle around an obstacle. During the evasion maneuver, the controller actively adjusts for threats such as wheel lift and tire skid using previewed state information. This research also presents key insights into the necessary preview horizons for effective intervention maneuvers.

1.3 Outline of Remaining Chapters

The remaining chapters in this thesis are organized as follows: Chapter 2 presents a review of the relevant literature on the topics of this thesis, including vehicle path following, collision avoidance, and rollover prevention. Naturally, many of the ideas reviewed in Chapter 2 use notation and concepts related to vehicle dynamics. Therefore, if the reader is unfamiliar with this subject, it is suggested that he/she read Chapter 3 first. Chapter 3 derives the vehicle models used in this thesis and defines standard vehicle dynamics notation. This includes 2- and 3-degree-of-freedom models with consideration of terrain and tire lag effects. Chapter 4 introduces the concept of the zero-moment point (ZMP) as a vehicle rollover metric. Previewed information about the ZMP is then explored and used to determine appropriate preview horizons for rollover prevention. Chapter 5 considers rollover prevention in a collision avoidance scenario with the purpose of determining the minimum intervention distance for a safe maneuver. Chapter 6 presents a linear-quadratic output regulator that safely maneuvers the vehicle around an obstacle while actively preventing rollover by weighting ZMP. The effects of ZMP weighting on the performance of the controller are also examined. Finally, Chapter 7 presents the conclusions made from this thesis and plans for future work.

Chapter 2 | Literature Review

2.1 Introduction

This chapter presents the relevant literature pertaining to autonomous vehicle control and rollover prevention. First, external research on vehicle path following and collision avoidance is summarized, with particular focus on strategies using previewed state information. Second, a review of vehicle rollover metrics and prevention strategies is presented. It should be noted that much of the research presented in this chapter is relevant in the aerospace field as well, for strategies to avoid aircraft collisions; however, only work pertaining to automotive control is covered.

2.2 Vehicle Path Following Review

An excellent resource on the use of optimal control theory in vehicle dynamics applications is presented by Sharp and Peng [4]. In [4], the authors provide a review of optimal control theories including the Linear-Quadratic Regulator (LQR), Linear-Quadratic Gaussian (LQG) theory, Nonlinear-Quadratic Regulator, indirect/direct optimization, model predictive control (MPC), and robust control. Literature applying these theories to vehicle dynamics problems is then discussed; specifically, the authors consider applications regarding active suspensions, worst-case maneuvering, minimum-time maneuvering, and driver modeling. Therefore, [4] provides a good starting point for the reader interested in reviewing optimal control practice within the vehicle dynamics community.

One of the most promising methods of vehicle path following is to utilize previewed information (what lies ahead of the vehicle) to predict vehicle behavior in the future. This framework has much in common with how a human driver operates; the vehicle strives to plan and execute a trajectory that not only reduces current error, but future error as well. Thus, the review provided in this chapter focuses mainly on control strategies utilizing previewed information.

2.2.1 Predictive Control Theory

The past decade has seen an explosion of research in the vehicle control field focusing on model predictive control (MPC) techniques. MPC is a finite-horizon optimal control structure in which plant dynamics are predicted over a fixed preview horizon to produce an optimal sequence of future control inputs [5]. This technique has proven to be a useful tool in path planning/following applications due to its ability to handle nonlinearities and system constraints.

Figure 2.1 illustrates the general process used in MPC in a vehicle control setting. The optimal control sequence is calculated by minimizing a user-defined objective function while also satisfying user-defined performance and input constraints. This calculation is iterative in nature; at every time step, the controller diagnoses the state of the system and calculates an optimal sequence of control inputs from the current time (k) up to the control horizon $(k + N_u)$. This sequence directs the vehicle along the optimal trajectory calculated up to the control horizon. Once the control sequence has been calculated, only



Figure 2.1. Illustration of model predictive control for vehicle path following. At each time step, an optimal trajectory and optimal control sequence are calculated. The vehicle executes the first control action and repeats the process.

the first input, corresponding to the control input at the current time step, is executed. The process is then repeated at the next time step (k+1) to produce a different optimal trajectory and control sequence.

The research field is rich with literature on MPC and general preview techniques for vehicle path planning and collision avoidance [6]- [33]. These strategies range from low complexity metrics that establish a time-to-line crossing [6], to higher complexity frameworks based on elastic bands for path planning through stationary and moving traffic [7]. Efforts have even been taken to combine haptic human-machine interaction with MPC-based algorithms [8]. For the sake of a reasonably succinct discussion, only a select few of the most relevant papers are subsequently discussed in detail throughout the rest of the chapter. An MPC framework becomes especially valuable in emergency situations; therefore, much of the discussion of work in this area is provided in Section 2.3.

Vehicle guidance and driver modeling using predictive control techniques is a relatively mature concept, with some of the earliest work proposed by MacAdam [9]. In [9],



Figure 2.2. Illustration of the MacAdam controller framework, which runs off of the error between the upcoming road trajectory (previewed input) and the predicted vehicle path (previewed output) over a specified preview horizon.

MacAdam developed a linear, time-invariant, and unconstrained predictive controller. Although this work can be viewed as a simplified subset of MPC techniques, it serves as a building block for many of the more advanced predictive control structures. The goal of MacAdam's controller is simple: to minimize the error between the previewed input (the road) and the previewed output (the predicted vehicle path), as seen in Fig. 2.2.

The previewed road input is assumed to come from mapped information or external measurements using sensors such as cameras or radar. The future state evolution that estimates the predicted vehicle path, meanwhile, is governed by the general solution of a linear dynamic system over a preview interval, T:

$$\vec{y}(t+T) = C(t)\Phi(t+T,t)\vec{x}(t) + \int_{t}^{t+T} C(\tau)\Phi(t+T,\tau)B(\tau)u(\tau)d\tau$$
(2.1)

where Φ is the system's state transition matrix, B is the system's input matrix, and C is the system's output matrix [9]. Equation 2.1 can then be simplified by assuming a time-invariant system and that the steering input remains constant over the preview interval.

Additionally, the model is subject to the following local performance index on the

vehicle's lateral position:

$$J \triangleq \frac{1}{T} \int_{t}^{t+T} \left\{ [f(\eta) - y(\eta)] W(\eta - t) \right\}^{2} d\eta$$
 (2.2)

where f is the previewed input, y is the previewed output, and W is an arbitrary weighting function over the preview interval. This structure seeks an optimal solution to Eq. 2.2, resulting in a steering input trajectory for different specifications of the weighting function. If the weighting function is specified such that only the point at the end of the preview horizon is considered, known as single point preview, the steering control input reduces to an intuitive proportional controller as such:

$$\delta_f(t) = \frac{f(t+T) - y(t+T)}{TK}$$
(2.3)

This control structure can be limited, however, by the fact that only a single point of previewed information is used, while no weight is given at the current time step. This results in a controller that prematurely steers the vehicle through the specified path, especially for severe maneuvers. Further, one has to predict into the future for a duration of T seconds, a process which assumes an accurate vehicle dynamic model.

In [10], Peng extended the MacAdam model with the goal of developing a model to represent a range of drivers with different characteristics. The following modifications were made to the MacAdam model: vehicle orientation errors were included in the cost function, the model was identified in real-time, and non-constant steering angle control was allowed over the preview horizon. Simulation results showed improved path following over the MacAdam model due to the inclusion of heading error and non-constant steering predictions.

2.2.2 Feedforward/Feedback Control with Preview

An alternative implementation of previewed information is to use projected road geometry to augment controller performance. Several variations of this strategy exist including a Proportional-Integral-Derivative (PID) controller developed by Taylor et al. [11] that combines projected lateral position error feedback with a projected road geometry feedforward term. Additionally, as part of the Partners for Advanced Transportation Technology (PATH) program, Peng and Tomizuka [12] combined an LQ feedback framework on lateral tracking error with two feedforward control terms: projected road curvature and projected road superelevation. Researchers on the PATH program then continued similar research in this area in the following years [13, 14].

Yet another implementation, similar to [12], was introduced by Sharp and Valtetsiotis [15]. In this work, the authors use previewed road information in a discrete LQR framework for path following. This idea is illustrated in Fig. 2.3, where both the road and vehicle are described by their lateral position and heading with respect to a global axis. The previewed road information then consists of a vector of road position points specified from the current time step (k) up to the preview horizon. At every time step, the previous road position is removed from this vector and a new road position $(y_{ri},$ treated as an input) is appended, creating a shift register operation:

$$\vec{y}_r(k+1) = D\vec{y}_r(k) + Ey_{ri}$$
(2.4)



Figure 2.3. Illustration of the road preview concept in [15]. The road and vehicle coordinates are described by their lateral position and heading with respect to a fixed axis.

where

$$D = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \qquad E = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$
(2.5)

The road preview vector is then combined with the state equations for the vehicle model to create an augmented state space model of the following form:

$$\begin{bmatrix} \vec{x}(k+1) \\ \vec{y}_r(k+1) \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix} \begin{bmatrix} \vec{x}(k) \\ \vec{y}_r(k) \end{bmatrix} + \begin{bmatrix} 0 \\ E \end{bmatrix} y_{ri} + \begin{bmatrix} B \\ 0 \end{bmatrix} \delta_f$$
(2.6)

where A and B are simply the discrete state and input matrices, respectively, of the vehicle model with state vector $\vec{x}(k)$ (the authors use a variant of the standard two-degree-of-freedom model derived in Chapter 3).

The reader may notice from Eq. 2.6 that there is no relationship between the vehicle dynamics and the road preview vector. The coupling between road motion and vehicle dynamics occurs in the cost function specified for the discrete LQR problem:

$$J = \lim_{n \to \infty} \sum_{k=0}^{n} [\vec{z}^{T}(k) R_{1} \vec{z}(k) + \delta_{f}(k) R_{2} \delta_{f}(k)]$$
(2.7)

where

$$\vec{z}(k) = \begin{bmatrix} \vec{x}(k) \\ \vec{y}_r(k) \end{bmatrix} \qquad R_1 = C^T Q C \qquad R_2 = 1$$
(2.8)

and

$$C = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{UT} & \frac{-1}{UT} & 0 & \dots & 0 \end{bmatrix} \qquad Q = \begin{bmatrix} q_y & 0 \\ 0 & q_\psi \end{bmatrix}$$
(2.9)

The Q matrix is arranged such that the controller penalizes the vehicle's lateral position deviation from the road and heading deviation from the road.

Minimization of Eq. 2.7 by solving the discrete algebraic Riccati equation for the augmented system of Eq. 2.6 results in an optimal preview gain vector. The closed-loop optimal steering input is then given by the following:

$$\delta_f(k) = -\vec{K}\vec{z}(k) = -[\vec{K}_1 \quad \vec{K}_2][\vec{x}(k) \quad \vec{y}_r(k)]^T$$
(2.10)

The vector \vec{K}_1 represents the control gain associated with the state feedback, non-preview LQR solution, while the vector \vec{K}_2 represents the preview control, i.e. how much steering input is allocated to road positions ahead of the vehicle's present position. This is an important insight into how this control structure differs from that of [9] and [10]. The authors do not predict the future state evolution of the vehicle, they simply augment the state space model with previewed road information. Results from [15] of this relatively simplistic model show excellent path following performance for a variety of road trajectories and weighting combinations. Further, because the vehicle motion is decoupled from future road position, there is not a need to predict vehicle dynamics in the future, nor is there a need to specify a single look-ahead point.

Research by Cole et al. [16] aimed to compare the predictive controllers of MacAdam [9], Peng [10], and Sharp [15], both in structure and in performance. In this work, an



Figure 2.4. Illustration depicting the difference between the preview horizon and the control horizon.

identical vehicle model was applied to controllers derived using unconstrained predictive control theory and LQ control theory. In particular, the authors examined how the choice of preview and control horizons affect the controller performance. The preview horizon refers to how far into the future information is acquired, whereas the control horizon refers to the horizon over which the objective function is calculated, or put another way, how far into the future the optimal control sequence is calculated (Fig. 2.4). Results showed that when the control and preview horizons are sufficiently long, the control gains derived from the different theories are identical. Deviations between the two theories occur when the control horizon is shorter than the preview horizon due to their different cost functions [16].

2.3 Vehicle Collision Avoidance Review

The vehicle path planning problem in the presence of threats consists of many different parts, especially when attempting to share control with a driver. Figure 2.5 illustrates a simplified architecture of how this problem is typically broken down [17]. The main elements of this architecture include a trajectory generator, a control system, and the vehicle/driver models.



Figure 2.5. Simplified architecture for a semi-autonomous vehicle guidance system.

The trajectory generator typically calculates an offline nominal trajectory that the vehicle will follow. This nominal trajectory, however, needs to be adjusted in real-time with feedback from the vehicle states due to tracking errors, disturbances, obstacles, etc. The control system module performs this function. The control system updates the vehicle trajectory and sends commands to the vehicle (along with the driver in a semi-autonomous vehicle).

As Fig. 2.5 shows, the control system can be broken down even further [18]. In a semi-autonomous system, a threat assessment layer actively monitors the projected trajectory, vehicle states, environment inputs, and driver behavior to assess the present threat to the vehicle, as well as in the near future. The control system then needs to decide if, when, and how much to intervene, as well as the most appropriate trajectory adjustment. The final step in the control system is to provide the necessary intervention, via a specified control law, to safely navigate the vehicle.

2.3.1 Threat Assessment

Determining when a vehicle must intervene in an emergency scenario is a difficult problem. Too much intervention can be seen as intrusive, while not enough intervention can result in an accident. Therefore, a very active research topic is not only how the vehicle will intervene, but rather assessing the threat to determine when the vehicle will intervene, especially in a semi-autonomous system. Jansson and Gustafsson approach this problem using statistical analysis [19]. Sensor uncertainty inevitably affects the decisions made by driver assist systems. The authors recognize this fact by formulating decision rules on when the vehicle should apply a braking intervention based on Bayesian collision probabilities. These risk probabilities are determined from stochastic sensor error models using Monte Carlo methods. The calculated risks are then combined with simple time-to-collision metrics to form decision rules. For the interested reader, Jansson provides a more complete discussion of these methods, including tracking models for multiple moving obstacles, in [20].

In [21], Falcone et al. introduce two model-based threat assessment methods. The overall idea is to specify a set of constraints and vehicle states/inputs that satisfy a safe trajectory. The threat assessment evolves over a future time horizon using a receding horizon predictive analysis, with the system checking whether the vehicle's current and near future states are within this safe set. This safe set is calculated using reachability analysis and set invariance theory; for example, the set must not contain states that intersect with stochastic reachable sets (unknown future positions) of other vehicles, pedestrians, etc. The two threat assessment methods then consist of 1) vehicle motion described by a vehicle model only and 2) vehicle motion is described by a driver and vehicle model.

2.3.2 Intervention

An early formulation of the constrained MPC problem, applied to vehicle path planning on low-friction roads (rain, snow, ice, etc.), appeared by Falcone et al. [17]. In this work two different controllers were derived, simulated, and tested: nonlinear MPC using a nonlinear vehicle and tire model and linear time-varying MPC using successive linearizations of the vehicle and tire model at each time step. The study found that the computation time for the nonlinear controller becomes unacceptable at moderate and high speeds (approximately 1.3 sec when traveling at 17 m/s). The linear time-varying controller, however, did not suffer from this drawback and provided acceptable tracking performance, even though the solution was suboptimal to that of the nonlinear optimization. A key insight found by the authors showed that constraining tire slip angle in the linear controller significantly enhanced the controller's performance, an intuitive result of keeping the tires away from highly nonlinear and possibly unstable regions. The paper then shows close agreement between simulation results of the two controllers and experimental data gathered on snow-covered roads. In [22], Falcone et al. extend this work by supplementing the linear time-varying MPC model with the additional control variables of braking and active differentials.

In [23], Madas et al. compared the the performance of linear MPC techniques against two other path planning methods for collision avoidance: a state lattice planner and a spline-based search tree algorithm. State lattice techniques create trajectories by connecting a grid of vertices with cubic spirals subject to vehicle kinematics. The splinebased search tree method uses a search tree to generate optimal paths that run tangent to all combinations of objects. Further discussion of these techniques is presented in [23].

The authors of [23] found that in collision avoidance simulations, all methods produced similar lateral position trajectories. Differences arose, however, in the lateral accelerations and lateral jerks of each path, implying different driver experiences during the maneuvers. A "best" solution was not provided; rather, the authors discuss the many trade-offs between the methods including ease of optimization/implementation, planning completeness, computational complexity, etc.

2.3.3 Combined Threat Assessment and Intervention

Anderson et al. [24] proposed a model predictive controller that combines the problems of trajectory planning, path following, and threat assessment in a collision avoidance scenario. The work in [24] navigates the vehicle through an optimized safe operating corridor by minimizing an objective cost function subject to lateral position and input constraints. The lateral position constraints require the vehicle to remain between maximum and minimum lateral position values along the road edge (or around an obstacle), while the input constraints define hard limits on the physical capabilities of the steering actuator.

The work in [24] also addresses the issue of driver interaction in semi-autonomous control by blending the driver input with the controller input. Thus, during low threat situations, this blending gain gives full control to the driver, while during high threat situations, full control is given to the controller. In order to provide a smooth control transition, the blending gain increases linearly between full driver control and fully autonomous. Two metrics were used to asses the instantaneous threat to the vehicle and subsequent level of intervention: the maximum value of tire slip angle over the prediction horizon, and a slightly more complicated quadratic cost function that penalizes large inputs and input rates in addition to tire slip angle. Tire slip angle was chosen as an output to minimize because small tire slip angles increase vehicle stability and result in a safer, more comfortable ride.

The authors [24] then test the controller performance using simulation and full-scale

experimental studies for a variety of collision avoidance scenarios. The results show that the controller is able to keep the vehicle in the safe operating corridor for all scenarios and threat functions; however, as expected, best performance is shown when the controller operates in fully autonomous mode. The controller is also able to avoid all obstacles and remain on the road while sharing steering with the driver. The authors extend this work in [25] by placing more emphasis on minimally-invasive trajectories.

Work by Gray et al. in [26, 27] posed a similar problem to that of [24] by using MPC to solve the threat assessment and intervention tasks as a combined optimization problem for a semi-autonomous vehicle. The main goal of this work was to determine the least intrusive intervention that will keep the vehicle in a safe operating state, only supplementing driver control to avoid safety constraint violations. In addition to steering control, the work in [26, 27] also includes braking as a control input.

The task of optimizing the least intrusive intervention is accomplished by treating lateral position deviation from the road as a safety constraint rather than a state that is optimized. Thus, the cost function in the constrained optimization problem does not include a penalty on tracking errors, but it is subject to the lateral position constraints. Additionally, a driver model is employed to predict what the driver's nominal behavior will be in the future. If the predicted trajectory based on nominal driver behavior does not satisfy the safety constraints, only then will the controller provide input; this ensures minimum intervention [26, 27].

One of the limitations seen in [26,27] was the assumption of a perfect driver model. In reality, drivers behave in unexpected and unpredictable ways. This issue was addressed by the authors in a follow-up paper [28], where an uncertain driver model was used. As a result, rather than one predicted trajectory, sets of predicted trajectories were calculated based on the spread of driver behavior. Robust MPC was then used to determine the least intrusive intervention to keep the vehicle and driver safe using a similar but modified structure to that of [26, 27].

Yet another strategy was proposed by Ali et al. [18], who built off the work in [17,22] by combining the MPC controller with a threat assessment layer. Based on the detected threat, the threat assessment layer decides the type of intervention the vehicle should impose to achieve the least intrusive controller that does not violate safety constraints. This ranges from a warning light for low-level threats (entering a curve too fast), to full autonomous MPC control for high-level threats (navigating a curve at high speeds on a low-friction surface).

2.4 Vehicle Roll Stability Review

Due to the high fatality rate of rollover incidents, investigating the roll stability of a vehicle is important in improving overall safety. This is especially true in emergency scenarios, such as collision avoidance, where the vehicle experiences conditions that increase rollover propensity.

2.4.1 Rollover Metrics

The most explicit method of analyzing a vehicle's roll characteristics is full-scale testing. While full-scale testing provides concrete results on vehicle behavior during worst-case maneuvers, it is expensive and incomplete. It is impossible to test all driving scenarios that could lead to wheel lift when considering factors such as speed, trajectory, vehicle type, weather conditions, etc. Additionally, the National Highway Traffic Safety Administration (NHTSA) currently uses a rollover rating based on physical vehicle parameters combined with an open-loop maneuver known as the "Fishhook" maneuver, where the vehicle is given an initial rapid steering input followed by an overcorrection [34]. While these tests provide important results, they do not represent the worse-case scenario for all vehicles under all conditions [4].

Therefore, much of the current research regarding roll stability aims to quantify a vehicle's rollover propensity by establishing model-based metrics. Many different rollover metrics exist including static or steady state metrics, dynamic metrics, metrics relying on ground-vehicle forces, and metrics utilizing vehicle state information.

A commonly used static metric is the Static Stability Factor [35]. The NHTSA rollover rating system mentioned above relies on a vehicle's SSF value, as well as dynamic testing and crash data. The SSF can be interpreted as the lateral acceleration necessary for wheel lift to occur on a flat road during steady state cornering [35]. By treating the vehicle as a rigid body, as seen in Fig. 2.6, the SSF is derived by performing a sum of moments about the right tire contact point as such:

$$\sum M_{RT} = ma_y h - mg \frac{T_r}{2} = 0$$
 (2.11)

Solving this moment balance and rearranging terms provides the SSF metric:

$$SSF = \frac{T_r}{2h} = \frac{a_y}{g} \tag{2.12}$$

where a_y is lateral acceleration, g is gravitational acceleration, T_r is the vehicle's track width, and h is the height of the center of gravity. The SSF is useful for a quick evaluation of rollover propensity using physical parameters; however, because it is based on a static moment balance, it cannot fully characterize roll stability during dynamic maneuvers.

More descriptive metrics that measure rollover propensity over a wide range of driving conditions fall into the category of dynamic metrics. Examples include the Dynamic Stability Index (DSI) [36], the Load Transfer Ratio (LTR) [37], and the Stability Moment (SM) [38]. The first of these metrics, the DSI, once again performs a moment balance about the right tire contact point of Fig. 2.6. However, now the vehicle's inertia and



Figure 2.6. Free-body diagram of a rigid vehicle model.

angular acceleration are considered as such:

$$\sum M_{RT} = ma_y h - mg \frac{T_r}{2} = I_{xx} \alpha_x \tag{2.13}$$

where I_{xx} is the vehicle's x-axis moment of inertia and α_x is the vehicle's angular acceleration about the x-axis. The vehicle is also assumed to be on the verge of wheel lift, meaning the forces acting on the left tire contact point are negligible. Solving the moment balance and rearranging terms yields the equation for the DSI:

$$DSI = \frac{T_r}{2h} = \frac{a_y}{g} - \frac{I_{xx}\alpha_x}{mgh}$$
(2.14)

Dynamic metrics can also be derived using vehicle-ground forces and moments. The LTR [37], proposed by Ervin at the University of Michigan Transportation Institute, is defined as a ratio: the difference in normal forces of the right and left tires divided by the sum of the normal forces in the right and left tires. This metric can be written as the following equation:

$$LTR = \frac{F_{zR} - F_{zL}}{F_{zR} + F_{zL}}$$
(2.15)

where F_{zR} is the right tire normal force and F_{zL} is the left tire normal force. Equation 2.15 indicates that the LTR can only vary between 1 and -1, with a value of 1 indicating wheel lift on the left side of the vehicle and a value of -1 indicating wheel lift on the right side of the vehicle.

A similar rollover metric which also uses information about vehicle-ground forces is the Stability Moment (SM) [38], proposed by Peters and Iagnemma. The SM is defined as the moment produced by the vehicle-ground contact forces about the tip-over axes of the vehicle, where the tip-over axes are the lines connecting the contact points of the tires. Rollover propensity is then measured by calculating the ratio of the stability moment difference to the stability moment sum between the left and right side of the vehicle. This metric can be written as the following equation:

$$R_{SM} = \frac{SM_L - SM_R}{SM_L + SM_R} \tag{2.16}$$

where SM_L and SM_R are the stability moments on the vehicle's left and right side, respectively. Once again, Eq. 2.16 indicates that this metric only varies between -1 and 1 with the same implications as those for LTR.

There are many additional rollover metrics used in the literature, including those used for tripped rollover [39]. Tripped rollover, although it is not considered in this thesis, refers to when a vehicle encounters some external tripping mechanism, such as a curb, with sufficient lateral velocity to cause rollover. A comprehensive list and summary of all types of rollover metrics is provided by Lapapong in [40].

2.4.2 Rollover Prevention with Preview

While predictive control approaches are very common for path following and collision avoidance applications, there is limited literature with regards to rollover prevention. In



Figure 2.7. Flowchart of the Time-to-Rollover (TTR) algorithm.

order to correct dangerous maneuvers and avoid future wheel lift, the vehicle needs to predict the impending roll threat. Several approaches implement previewed information in the design of active suspensions for rollover prevention [33,41,42]. Since this thesis does not investigate active suspensions, however, the following discussion will focus mainly on steering- and braking-based approaches.

One of the earlier predictive rollover metrics introduced by Chen and Peng was the Time-to-Rollover (TTR) metric [43, 44]. The predicted TTR is defined as the time it takes for the vehicle's sprung mass to reach a critical, user-defined roll angle with respect to the unsprung mass. Using the vehicle's current state and assuming constant steering and speed, this calculation integrates the vehicle model up to the end of the prediction horizon. If the vehicle's roll angle passes the threshold, the TTR is established. Otherwise, the model recalculates at the next time step, as illustrated in Fig. 2.7.

The TTR metric is assessed by the authors in [44] with focus on model fidelity; simple models offer real-time calculation while complex models offer a more accurate TTR estimate. Specifically, a neural network (NN) approach is used to balance the trade-off between the simple and complex models to generate a new NN-TTR metric with increased accuracy. In [43], Chen and Peng use TTR to design a controller that prevents rollover using differential braking. Differential braking has several benefits when considering control inputs for rollover prevention: 1) it is much less expensive than active suspension/stabilizer systems and can be easily implemented on existing vehicles. 2) it is considered to be a highly effective way to reduce lateral acceleration by simply saturating the tire's force capability in the longitudinal direction, thus reducing the tire's ability to produce lateral forces. 3) it can reduce the vehicle's speed as well. The authors then continue to provide simulation results of a vehicle's roll threat (quantified using the LTR) with and without the TTR-based controller.

Yu et al. [42] extended the use of the TTR metric towards heavy-duty vehicles applications and claimed two improved variants of the TTR calculation based on a higher-order vehicle model. The first variant assumes constant steering angle and constant vehicle acceleration (rather than vehicle speed) in the future. This improves the TTR prediction for maneuvers that experience large speed variations, such as braking. The second variant assumes constant steering angular acceleration and constant vehicle acceleration in the future. By assuming constant steering angular acceleration, the TTR prediction becomes more aggressive, meaning it detects rollover-prone situations earlier, but also increases the risk of false alarms. The authors then compare all three TTR calculations in the performance of an active suspension predictive LQR controller.

Another approach to predictive rollover prevention was proposed by Carlson and Gerdes [45]. This controller adopts an MPC framework to perform yaw rate tracking while satisfying the constraint that the vehicle's roll angle cannot pass a user-defined threshold. The authors consider the two control inputs to be steering and differential braking. Ultimately, this results in a controller that understeers the vehicle to prevent rollover, but also accommodates yaw rate tracking through differential braking-induced moments. Two different control laws are then derived and simulated: one which assumes complete knowledge of the driver's input in advance, and one that assumes no knowledge of the driver's input in the future and thus holds the current inputs constant over the preview horizon.

Similarly, work by Schofield and Hagglund [46] adopted an MPC-based approach to limit the vehicle's peak roll angle by using differential braking as the control action. Rather than simply applying differential braking commands to individual wheels as in [45], the authors implement a framework that optimally allocates control to the four wheel simultaneously. This takes advantage of all available actuators to produce optimized forces and moments that mitigate the rollover threat.

2.4.3 Zero-Moment Point

Many of the metrics discussed above are useful under certain conditions but can be limited by their inherent assumptions. Static metrics such as the SSF are easy to calculate, but do not capture vehicle behavior during dynamic maneuvers. Meanwhile, dynamic metrics such as the LTR and SM rely on information that is difficult to obtain. Sensors capable of measuring the forces and moments acting on the tires are expensive and uncommon on passenger vehicles. Finally, metrics that use state prediction models such as the TTR do not account for environmental factors such as terrain. Terrain effects greatly influence a vehicle's rollover propensity and are often ignored in the literature. Several of these metrics also saturate at the onset of wheel lift, meaning they are unable to predict the severity of certain maneuvers beyond this point.

To address these limitations, Lapapong applied the concept of the zero-moment point (ZMP) as a vehicle rollover metric [40, 47, 48]. The ZMP is defined as the point on the ground where the summation of tipping moments, due to gravity and inertia forces, equals zero [49]. The concept surrounding the ZMP is that roll stability analysis can be performed using inertial responses rather than force measurements. Vukobratovic originally introduced this idea [50] and it has been widely used to maintain the dynamic
stability of bipedal robots. The concept of the ZMP as it relates to vehicle rollover is further discussed in Chapter 4, including a full derivation.

When applied as a vehicle rollover metric, the ZMP presents several advantages over the metrics discussed above. First, the ZMP explicitly accounts for terrain effects in its derivation, a feature that is extremely useful when evaluating worst-case scenarios for rollover. The inclusion of terrain effects results in a more descriptive rollover metric when compared to other metrics such as the DSI, which is also derived through a moment balance. Second, the ZMP can be expressed as a linear combination of the vehicle's states, allowing it to be included as an output of any vehicle model. For these reasons, the ZMP was chosen as the rollover metric used in this work.

Lapapong's work has proven that, as a design parameter, the ZMP is an accurate indication of rollover [48]. It has not, however, been applied in a predictive manner with the intention of preventing future rollover in a collision avoidance scenario using closed-loop control. This is the focus of this thesis.

Chapter 3 Derivation of Vehicle Models

3.1 Introduction

This chapter derives the dynamic vehicle models and nomenclature used throughout the rest of the thesis. First, a two-degree-of-freedom (2DOF) rigid model is presented. Section 3.3 then derives a three-degree-of-freedom (3DOF) vehicle model that includes roll dynamics. These models are then extended in Sections 3.2.1 and 3.3.1 to account for terrain effects. Finally, these models are further extended in Sections 3.2.2 and 3.3.2 to account for tire lag dynamics.

All of the models derived in this chapter are linear in nature. These descriptive, but relatively simplistic, vehicle models facilitate exhaustive simulation of various maneuvers. In addition, they can ultimately be implemented on a vehicle-borne microprocessor for vehicle control.



Figure 3.1. SAE body-fixed vehicle coordinate system.

3.2 Two-Degree-of-Freedom "Bicycle Model"

One of the most commonly used models in the vehicle dynamics community is a 2DOF model commonly referred to as the bicycle model.¹ The bicycle model only considers the vehicle's lateral and yaw dynamics. The body-fixed coordinate system used in the derivation follows the Society of Automotive Engineers (SAE) convention [51]. This coordinate system defines the positive body-fixed x, y, and z directions to point forward, right, and down respectively, as shown in Fig 3.1. An illustration of the free-body diagram for the bicycle model is shown in Fig 3.2 and the nomenclature used in the derivation is presented in Table 3.1.

The bicycle model relies on the following key assumptions that aid in the derivation:

- The vehicle is assumed to be symmetrical along its longitudinal axis.
- No motion exists in the roll and pitch directions.

 $^{^{1}}$ It should be noted that the naming of this model is one of convention, it does not actually describe the dynamics of a bicycle.



Figure 3.2. Free-body diagram of bicycle model in body-fixed coordinates.

Table	3.1.	Parameters an	id nomenclature	for th	e bicycle	e model.	Nominal	values	used	in a	simu-
lation	are p	resented in Sec	tion 3.4 .								

Parameter	Definition		
U	Longitudinal velocity of CG (body-fixed)	m/s	
y	Lateral position of vehicle	m	
V	Lateral velocity of CG (body-fixed)	m/s	
ψ	Yaw angle (heading) with respect to a global frame	rad	
r	Yaw rate	rad/s	
m	Vehicle mass	kg	
I_{zz}	Mass moment of inertia about vehicle z-axis	kg-m ²	
a	Distance from CG to front axle along vehicle x-axis	m	
b	Distance from CG to rear axle along vehicle x-axis	m	
L	Length of vehicle $(a + b)$	m	
T_r	Track width of vehicle	m	
F_{f}	Front tire force	Ν	
F_r	Rear tire force	N	
$C_{lpha f}$	Front tire cornering stiffness	N/rad	
$C_{lpha r}$	Rear tire cornering stiffness	N/rad	
α_f	Front tire slip angle	rad	
α_r	Rear tire slip angle	rad	
δ_f	Front steering angle	rad	

- The vehicle is steered by the front wheels.
- The vehicle has a constant longitudinal velocity, U.
- Small angle approximations apply such that $\sin(\theta) \approx \theta$ and $\cos(\theta) \approx 1$.
- A linear tire model is applied such that F = C_αα; thus, the lateral force on the tire is linearly proportional to the tire slip angle.
- The tires are assumed to roll without slipping in the longitudinal direction.
- Aerodynamic effects are negligible.

Using the SAE body-fixed coordinate system described previously, the angular ve-

locity, $\vec{\omega}$, and angular acceleration, $\vec{\alpha}$, of the vehicle can be written as the following:

$$\vec{\omega} = r\hat{k} \tag{3.1}$$

$$\vec{\alpha} = \dot{r}\hat{k} \tag{3.2}$$

The linear velocity of the vehicle's center of gravity, meanwhile, can be expressed as

$$\vec{v}_o = U\hat{i} + V\hat{j} \tag{3.3}$$

To apply Newton's equations, however, the accelerations of the vehicle must be written with respect to a global, fixed-frame coordinate system. The fixed-frame acceleration of a moving object can be written by the general equation

$$\vec{a} = \dot{\vec{v}}_{o,moving} + \vec{\omega} \times \vec{v}_{o,moving} \tag{3.4}$$

Solving this equation using Eq. 3.1 and Eq. 3.3 yields the following for the acceleration of the vehicle in body-fixed coordinates:

$$\vec{a} = \dot{U}\hat{i} + \dot{V}\hat{j} + Ur\hat{j} - Vr\hat{i} \tag{3.5}$$

$$\vec{a} = (-Vr)\hat{i} + (\dot{V} + Ur)\hat{j}$$
 (3.6)

where $\dot{U} = 0$ due to the constant velocity of the vehicle in the body-fixed longitudinal direction. Referring back to Fig. 3.2, a sum of forces in the lateral direction and a sum of moments about the z-axis yield the following equations of motion for the bicycle model:

$$\Sigma F_y = ma_y = m(\dot{V} + Ur) = F_f + F_r \tag{3.7}$$

$$\Sigma M_z = I_{zz} \dot{r} = aF_f - bF_r \tag{3.8}$$

Applying the assumption of the linear tire model, the front and rear tire forces can be expressed as $F_f = C_{\alpha f} \alpha_f$ and $F_r = C_{\alpha r} \alpha_r$ respectively. Substituting these forces into Eq. 3.7 and Eq. 3.8 yields the following:

$$m(\dot{V} + Ur) = C_{\alpha f} \alpha_f + C_{\alpha r} \alpha_r \tag{3.9}$$

$$I_{zz}\dot{r} = aC_{\alpha f}\alpha_f - bC_{\alpha r}\alpha_r \tag{3.10}$$

The geometry of Fig. 3.2 allows the tire slip angles to be written as functions of the tire velocities and the front steering angle in the form

$$\alpha_f = \arctan\left(\frac{v_{fy}}{v_{fx}}\right) - \delta_f \approx \left(\frac{v_{fy}}{v_{fx}}\right) - \delta_f \approx \frac{V + ar}{U} - \delta_f \tag{3.11}$$

$$\alpha_r = \arctan\left(\frac{v_{ry}}{v_{rx}}\right) \approx \frac{v_{ry}}{v_{rx}} \approx \frac{V - br}{U}$$
(3.12)

Small angle approximations assume that the inverse tangent function is approximately equal to the angle itself. Substituting Eq. 3.11 into Eq. 3.9 and Eq. 3.12 into Eq. 3.10 yields the final equations of motion for the bicycle model.

$$m(\dot{V} + Ur) = C_{\alpha f} \left(\frac{V + ar}{U} - \delta_f\right) + C_{\alpha r} \left(\frac{V - br}{U}\right)$$
(3.13)

$$I_{zz}\dot{r} = aC_{\alpha f} \left(\frac{V+ar}{U} - \delta_f\right) - bC_{\alpha r} \left(\frac{V-br}{U}\right)$$
(3.14)

Rearranging to solve for \dot{V} and \dot{r} produces the following:

$$\dot{V} = \left(\frac{C_{\alpha f} + C_{\alpha r}}{mU}\right)V + \left(\frac{aC_{\alpha f} - bC_{\alpha r}}{mU} - U\right)r - \left(\frac{C_{\alpha f}}{m}\right)\delta_f \tag{3.15}$$

$$\dot{r} = \left(\frac{aC_{\alpha f} - bC_{\alpha r}}{I_{zz}U}\right)V + \left(\frac{a^2C_{\alpha f} + b^2C_{\alpha r}}{I_{zz}U}\right)r - \left(\frac{aC_{\alpha f}}{I_{zz}}\right)\delta_f$$
(3.16)

Finally, Eq. 3.15 and Eq. 3.16 can be represented in matrix notation. This produces a state space model with states of V and r as such:

$$\begin{bmatrix} \dot{V} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{C_{\alpha f} + C_{\alpha r}}{mU} & \frac{aC_{\alpha f} - bC_{\alpha r}}{mU} - U \\ \frac{aC_{\alpha f} - bC_{\alpha r}}{I_{zz}U} & \frac{a^2C_{\alpha f} + b^2C_{\alpha r}}{I_{zz}U} \end{bmatrix} \begin{bmatrix} V \\ r \end{bmatrix} + \begin{bmatrix} \frac{-C_{\alpha f}}{m} \\ \frac{-aC_{\alpha f}}{I_{zz}} \end{bmatrix} \delta_f$$
(3.17)

If the lateral position and heading of the vehicle are of interest, the state space model of Eq. 3.17 can be modified to include these states with a state vector of the form

$$\vec{x} = \begin{bmatrix} y \\ V \\ r \\ \psi \end{bmatrix}$$
(3.18)

The A and B matrices of the state space model then become

$$A = \begin{bmatrix} 0 & 1 & 0 & U \\ 0 & \frac{C_{\alpha f} + C_{\alpha r}}{mU} & \frac{aC_{\alpha f} - bC_{\alpha r}}{mU} - U & 0 \\ 0 & \frac{aC_{\alpha f} - bC_{\alpha r}}{I_{zz}U} & \frac{a^2C_{\alpha f} + b^2C_{\alpha r}}{I_{zz}U} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$
(3.19)

$$B = \begin{bmatrix} 0\\ \frac{-C_{\alpha f}}{m}\\ \frac{-aC_{\alpha f}}{I_{zz}}\\ 0 \end{bmatrix}$$
(3.20)

3.2.1 Bicycle Model with Terrain Input

In many cases, the road profile is not perfectly flat in the lateral direction. This is also true just outside the road boundaries, as the terrain typically slopes down as it moves away from the road. Therefore, it is necessary to take the road bank angle into consideration for many driving scenarios. The terrain angle, ϕ_t , as seen in Fig. 3.3, is now considered when developing the equations of motion.

For the 2DOF bicycle model with small angle approximations, the terrain angle affects the equations of motion (Eq. 3.7 and Eq. 3.8) in the following manner:

$$\Sigma F_y = F_f + F_r + mg\sin(\phi_t) = F_f + F_r + mg\phi_t \tag{3.21}$$

$$\Sigma M_z = I_{zz} \dot{r} = aF_f - bF_r \tag{3.22}$$

Only the sum of forces along the vehicle's y-axis is affected by the inclusion of terrain; it has no contribution towards the sum of moments about the vehicle's z-axis.

To preserve linearity, the terrain angle is treated as a linear input to the system dynamics. Although this "input" cannot be controlled, it can be treated as known with



Figure 3.3. Bicycle model on a banked slope.

mapped information of the vehicle's surroundings. The equations of motion with bank angle input result in the following state space representation of the bicycle model:

$$\begin{bmatrix} \dot{V} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{C_{\alpha f} + C_{\alpha r}}{mU} & \frac{aC_{\alpha f} - bC_{\alpha r}}{mU} - U \\ \frac{aC_{\alpha f} - bC_{\alpha r}}{I_{zz}U} & \frac{a^2C_{\alpha f} + b^2C_{\alpha r}}{I_{zz}U} \end{bmatrix} \begin{bmatrix} V \\ r \end{bmatrix} + \begin{bmatrix} \frac{-C_{\alpha f}}{m} & g \\ \frac{-aC_{\alpha f}}{I_{zz}} & 0 \end{bmatrix} \begin{bmatrix} \delta_f \\ \phi_t \end{bmatrix}$$
(3.23)

The terrain bank angle is typically defined in units of degrees or as a percent superelevation. When defined as a percent superelevation, this paper uses the following equation:

$$\phi_{t,percent} = 100 * \tan\left(\phi_{t,deq}\right) \tag{3.24}$$

3.2.2 Bicycle Model with Tire Lag Dynamics

An assumption of the models above was that the tires are able to generate lateral force instantly from changes in steering input. In realistic driving situations, however, this is not true; there is lag between the steering input and force generation due to deformation of the tire sidewall. Tire lag can affect the roll characteristics of a vehicle and is important to model.

Tire lag is most commonly modeled as a first-order differential equation [52] in the form

$$\dot{F}_f = \frac{U}{\sigma_f} (F_{ss,f} - F_f) \tag{3.25}$$

$$\dot{F}_r = \frac{U}{\sigma_r} (F_{ss,r} - F_r) \tag{3.26}$$

where $F_{ss,f} = C_{\alpha f} \alpha_f$ and $F_{ss,r} = C_{\alpha r} \alpha_r$ are the steady-state tire forces and σ_f , σ_r are the front and rear tire relaxation lengths, respectively. Substituting $F_{ss,f}$, $F_{ss,r}$, and the slip angles defined in Eq. 3.11 and Eq. 3.12 we obtain the following:

$$\dot{F}_f = \frac{U}{\sigma_f} \left[C_{\alpha f} \left(\frac{V + ar}{U} - \delta_f \right) - F_f \right]$$
(3.27)

$$\dot{F}_r = \frac{U}{\sigma_r} \left[C_{\alpha r} \left(\frac{V - br}{U} \right) - F_r \right]$$
(3.28)

Inclusion of tire lag dynamics is done by adding the front and rear tire forces as states in the vehicle model. Using Eq. 3.7 and Eq. 3.8, with the addition of Eq. 3.27 and Eq. 3.28, results in the augmented state space model

$$\begin{bmatrix} \dot{V} \\ \dot{r} \\ \dot{F}_{f} \\ \dot{F}_{f} \\ \dot{F}_{r} \end{bmatrix} = \begin{bmatrix} 0 & -U & \frac{1}{m} & \frac{1}{m} \\ 0 & 0 & \frac{a}{I_{zz}} & \frac{-b}{I_{zz}} \\ \frac{C_{\alpha f}}{\sigma_{f}} & \frac{aC_{\alpha f}}{\sigma_{f}} & \frac{-U}{\sigma_{f}} & 0 \\ \frac{C_{\alpha r}}{\sigma_{r}} & \frac{-bC_{\alpha r}}{\sigma_{r}} & 0 & \frac{-U}{\sigma_{r}} \end{bmatrix} \begin{bmatrix} V \\ r \\ F_{f} \\ F_{r} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{-UC_{\alpha f}}{\sigma_{f}} \\ 0 \end{bmatrix} \delta_{f}$$
(3.29)

When bank angle input is also included, the model given in Eq. 3.29 becomes

$$\begin{bmatrix} \dot{V} \\ \dot{r} \\ \dot{F}_{f} \\ \dot{F}_{f} \\ \dot{F}_{r} \end{bmatrix} = \begin{bmatrix} 0 & -U & \frac{1}{m} & \frac{1}{m} \\ 0 & 0 & \frac{a}{I_{zz}} & \frac{-b}{I_{zz}} \\ \frac{C_{\alpha f}}{\sigma_{f}} & \frac{aC_{\alpha f}}{\sigma_{f}} & \frac{-U}{\sigma_{f}} & 0 \\ \frac{C_{\alpha r}}{\sigma_{r}} & \frac{-bC_{\alpha r}}{\sigma_{r}} & 0 & \frac{-U}{\sigma_{r}} \end{bmatrix} \begin{bmatrix} V \\ r \\ F_{f} \\ F_{r} \end{bmatrix} + \begin{bmatrix} 0 & g \\ 0 & 0 \\ \frac{-UC_{\alpha f}}{\sigma_{f}} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{f} \\ \phi_{t} \end{bmatrix}$$
(3.30)

3.3 Three-Degree-of-Freedom "Roll Model"

Although the 2DOF bicycle model is useful for understanding the lateral and yaw accelerations of the vehicle, it does not provide any information about the vehicle's roll dynamics. This section introduces a 3DOF roll model mathematically consistent with the vehicle model developed by Mammar [53], with the exception of the coordinate system location. This model is considered an extension of the bicycle model to understand the vehicle's roll dynamics.

An illustration of roll model and its coordinate system is shown in Fig. 3.4, while the nomenclature used in the derivation is shown in Table 3.2. The equations of motion



Figure 3.4. Free-body diagram of roll model in body-fixed coordinates.

are derived and written in mass-damper-spring form, then converted to a state space representation.

The roll model relies on all of the assumptions listed in Section 3.2 for the bicycle model with the addition of the following:

- No motion exists in the pitch direction (roll motion is now allowed).
- The torsional spring and torsional damper acting at the roll center are linear.
- The roll center is fixed with respect to the vehicle's body.
- The unsprung mass only rotates about the vehicle z-axis.

Referring to Fig. 3.4, the roll model can be described as dividing the vehicle mass into a sprung mass, G_s , and an unsprung mass, G_u . The sprung mass represents the mass of the vehicle that sits on top of the suspension, while the unsprung mass represents the mass of the vehicle that is located under the suspension. These two masses are connected at a joint called the roll center, which is defined as a virtual point about which the sprung mass rolls with respect to the unsprung mass. A torsional spring and torsional damper

Table 3.2. Parameters and nomenclature for the roll model. Nominal values used in simulationare presented in Section 3.4.

Parameter	Definition	Unit
U	Longitudinal velocity of CG (body-fixed)	m/s
y	Lateral position of vehicle	m
V	Lateral velocity of CG (body-fixed)	m/s
ψ	Yaw angle (heading) with respect to a global frame	rad
r	Yaw rate of vehicle	rad/s
ϕ_r	Roll angle of sprung mass	rad
θ	Pitch angle of vehicle	rad
m	Total vehicle mass	kg
m_u	Unsprung vehicle mass	kg
m_s	Sprung vehicle mass	kg
I_{xx}	Mass moment of inertia about vehicle x-axis	kg-m ²
Izz	Mass moment of inertia about vehicle z-axis	kg-m ²
I_{xz}	Product mass moment of inertia	kg-m ²
a	Distance from CG to front axle along vehicle x-axis	m
b	Distance from CG to rear axle along vehicle x-axis	m
L	Length of vehicle $(a + b)$	m
T_r	Track width of vehicle	m
h_r	Height of the roll center	m
h_s	Height of sprung mass CG	m
h_{sr}	Height of sprung mass from roll center	m
F_{f}	Front tire force	N
F_r	Rear tire force	N
$C_{lpha f}$	Front tire cornering stiffness	N/m
$C_{lpha r}$	Rear tire cornering stiffness	N/m
α_f	Front tire slip angle	rad
α_r	Rear tire slip angle	rad
δ_f	Front steering angle	rad
K_{ϕ}	Roll stiffness	N-m/rad
D_{ϕ}	Roll damping constant	N-m-s/rad
g	Gravitational acceleration	m/s^2

have been included at the roll center to simulate the vehicle's suspension dynamics. As indicated in the model assumptions, the roll center only allows rotation about the vehicle's body-fixed x-axis. For this derivation, the vehicle coordinate system is attached to the unsprung mass.

Considering the coordinate system shown in Fig. 3.4, the angular velocity of the

unsprung mass, $\vec{\omega}_u$, can be written as

$$\vec{\omega}_u = r\hat{k} \tag{3.31}$$

while the angular velocity of the sprung mass, $\vec{\omega}_s$, can be expressed as

$$\vec{\omega}_s = \dot{\phi}_r \hat{i} + r\hat{k} \tag{3.32}$$

Similarly, the linear velocity of the unsprung mass, \vec{v}_u , can be written as

$$\vec{v}_u = U\hat{i} + V\hat{j} \tag{3.33}$$

while the linear velocity of the sprung mass, \vec{v}_s , can be expressed as

$$\vec{v}_s = U\hat{i} + (V + h_{sr}\dot{\phi}_r)\hat{j} \tag{3.34}$$

Equation 3.34 was derived utilizing relative velocity and assuming a small roll angle. To determine the accelerations of the sprung and unsprung masses, the velocities must be converted to a global, fixed-frame coordinate system. This allows Newton's equations to be used to describe the system. The fixed-frame acceleration of a moving object can be written by the general equation

$$\vec{a} = \dot{\vec{v}}_{o,moving} + \vec{\omega} \times \vec{v}_{o,moving}$$
(3.35)

where \vec{a} is the fixed-frame acceleration of the object. Solving this equation for the unsprung mass and assuming the roll center is coincident with the unsprung mass's CG

yields the following equation:

$$\vec{a}_u = \dot{U}\hat{i} + \dot{V}\hat{j} + Ur\hat{j} - Vr\hat{i}$$
(3.36)

$$\vec{a}_u = (-Vr)\hat{i} + (\dot{V} + Ur)\hat{j}$$
 (3.37)

where $\dot{U} = 0$ due to the constant velocity of the vehicle in the body-fixed longitudinal direction. The acceleration of the sprung mass can be calculated as

$$\vec{a}_{s} = \dot{U}\hat{i} + (\dot{V} + h_{sr}\ddot{\phi}_{r})\hat{j} - r(V + h_{sr}\dot{\phi}_{r})\hat{i} + Ur\hat{j} + V\dot{\phi}_{r}\hat{k}$$
(3.38)

$$\vec{a}_{s} = -r(V + h_{sr}\dot{\phi}_{r})\hat{i} + (\dot{V} + Ur + h_{sr}\ddot{\phi}_{r})\hat{j} + V\dot{\phi}_{r}\hat{k}$$
(3.39)

When compared to the bicycle model, the addition of roll dynamics adds another equation of motion by performing a sum of moments about the vehicle's x-axis. Therefore, the three equations are obtained through a sum of forces along the vehicle's y-axis, a sum of moments about the vehicle's z-axis, and a sum of moments about the vehicle's x-axis. Summing the forces along the vehicle's y-axis yields the following equation:

$$\Sigma F_y = m_u a_{y,u} + m_s a_{y,s} = F_f + F_r \tag{3.40}$$

$$m(\dot{V} + Ur) + m_s h_{sr} \ddot{\phi}_r = F_f + F_r \tag{3.41}$$

The next equation is obtained by summing the moments about the roll center in the x-direction. Assuming the vehicle's roll center is close to the ground, this equation of motion can be written as

$$\Sigma M_{x,RC} = I_{xx}\ddot{\phi}_r - I_{xz}\dot{r} + m_s \left((-h_{sr}\hat{k}) \times \vec{a}_s \right) \cdot \hat{i}$$
(3.42)

$$I_{xx}\ddot{\phi}_r - I_{xz}\dot{r} + m_s h_{sr}(\dot{V} + Ur + h_{sr}\ddot{\phi}_r) = -D_{\phi}\dot{\phi}_r + (m_s h_{sr}g - K_{\phi})\phi_r$$
(3.43)

The last equation of motion is developed by summing the moments about the sprung mass in the z-direction to produce the following:

$$\Sigma M_{z,s} = I_{zz}\dot{r} - I_{xz}\ddot{\phi}_r + m_s\left((-h_{sr}\hat{k})\times\vec{a}_s\right)\cdot\hat{k} = aF_f - bF_r \tag{3.44}$$

$$I_{zz}\dot{r} - I_{xz}\ddot{\phi}_r = aF_f - bF_r \tag{3.45}$$

Equations 3.41, 3.43, and 3.45 are the three equations of motion for the roll model. These can be organized in the standard mass-damper-spring (MDK) form of

$$M\ddot{\vec{q}} + D\dot{\vec{q}} + K\vec{q} = F\vec{u} \tag{3.46}$$

where

$$\vec{q} = \begin{bmatrix} y \\ \psi \\ \phi_r \end{bmatrix}$$
(3.47)

defines the states of the MDK equation. These are the three degrees-of-freedom of the roll model: y is the lateral position, ψ is the yaw angle, and ϕ_r is the roll angle. By rearranging the three equations of motion, the mass, damper, and spring matrices become

$$M = \begin{bmatrix} m & 0 & m_s h_{sr} \\ m_s h_{sr} & -I_{xz} & I_{xx} + m_s h_{sr}^2 \\ 0 & I_{zz} & -I_{xz} \end{bmatrix}$$
(3.48)

$$D = \begin{bmatrix} 0 & mU & 0 \\ 0 & m_s h_{sr} U & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
(3.49)
$$K = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & K_{\phi} - m_s h_{sr} g \\ 0 & 0 & 0 \end{bmatrix}$$
(3.50)

The input to the model is defined as the front and rear tire forces in the lateral direction such that

$$\vec{u} = \begin{bmatrix} F_f \\ F_r \end{bmatrix}$$
(3.51)

and the force matrix is defined as

$$F = \begin{bmatrix} 1 & 1 \\ a & -b \\ 0 & 0 \end{bmatrix}$$
(3.52)

The equations of motion for the roll model can also be represented in state space form, facilitating numerical simulation of the system. The state space form can be derived from the MDK form by first rewriting the lateral forces of the system. Under the linear tire model assumption, these forces are defined as $F_f = C_{\alpha f} \alpha_f$ and $F_r = C_{\alpha r} \alpha_r$. As shown in Section 3.2, the tire slip angles can be written as functions of the velocities of each and front steering angle as follows:

$$\alpha_f = \arctan\left(\frac{v_{fy}}{v_{fx}}\right) - \delta_f \approx \left(\frac{v_{fy}}{v_{fx}}\right) - \delta_f \approx \frac{V + ar}{U} - \delta_f \tag{3.53}$$

$$\alpha_r = \arctan\left(\frac{v_{ry}}{v_{rx}}\right) \approx \frac{v_{ry}}{v_{rx}} \approx \frac{V - br}{U}$$
(3.54)

Substituting Eq. 3.53 and Eq. 3.54 in the lateral force equations and rearranging produces three equations of motion in the following form:

$$m\dot{V} + m_s h_{sr} \ddot{\phi}_r + \left(mU + \frac{bC_{\alpha r} - aC_{\alpha f}}{U}\right)r - \left(\frac{C_{\alpha f} + C_{\alpha r}}{U}\right)V = -C_{\alpha f}\delta_f \qquad (3.55)$$

$$(I_{xx} + m_s h_{sr}^2)\ddot{\phi}_r + m_s h_{sr}\dot{V} - I_{xz}\dot{r} + m_s h_{sr}Ur + D_\phi\dot{\phi}_r - (m_s h_{sr}g - K_\phi)\phi_r = 0 \quad (3.56)$$

$$I_{zz}\dot{r} - I_{xz}\ddot{\phi}_r - \left(\frac{a^2C_{\alpha f} + b^2C_{\alpha r}}{U}\right)r + \left(\frac{bC_{\alpha r} - aC_{\alpha f}}{U}\right)V = -aC_{\alpha f}\delta_f \tag{3.57}$$

By introducing a fourth equation (which adds ϕ_r as a vehicle state),

$$\dot{\phi}_r = \dot{\phi}_r \tag{3.58}$$

an intermediate MDK model can be introduced in the form

$$M_{int}\dot{\vec{x}} + N_{int}\vec{x} = F_{int}\delta_f \tag{3.59}$$

The state vector of this intermediate model, and ultimately the state space model, is then defined as

$$\vec{x} = \begin{bmatrix} V \\ r \\ \dot{\phi}_r \\ \dot{\phi}_r \end{bmatrix}$$
(3.60)

Writing the equations of motion in the form of Eq. 3.59 results in the following matrices:

$$M_{int} = \begin{bmatrix} m & 0 & m_s h_{sr} & 0 \\ m_s h_{sr} & -I_{xz} & I_{xx} + m_s h_{sr}^2 & 0 \\ 0 & I_{zz} & -I_{xz} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
(3.61)
$$N_{int} = \begin{bmatrix} \frac{-C_{\alpha f} - C_{\alpha r}}{U} & mU + \frac{bC_{\alpha r} - aC_{\alpha f}}{U} & 0 & 0 \\ 0 & m_s h_{sr} U & D_{\phi} & K_{\phi} - m_s h_{sr} g \\ \frac{bC_{\alpha r} - aC_{\alpha f}}{U} & \frac{-a^2 C_{\alpha f} - b^2 C_{\alpha r}}{U} & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$
(3.62)

The input to the intermediate model is the front steering angle, δ_f , with a force matrix of

$$F_{int} = \begin{bmatrix} -C_{\alpha f} \\ 0 \\ -aC_{\alpha f} \\ 0 \end{bmatrix}$$
(3.63)

The general state space form of

$$\dot{\vec{x}} = A\vec{x} + B\vec{u} \tag{3.64}$$

with a state vector defined in Eq. 3.60, can be obtained from this intermediate form through the following relationship:

$$A = -M_{int}^{-1}N_{int} (3.65)$$

$$B = M_{int}^{-1} F_{int} \tag{3.66}$$

If the lateral position and heading of the vehicle are of interest, the A and B matrices can be modified to include these states with a state vector of the form

$$\vec{x} = \begin{bmatrix} y \\ V \\ r \\ \dot{\phi}_r \\ \phi_r \\ \psi \end{bmatrix}$$
(3.67)

The A and B matrices of the state space model then become

$$A = \begin{bmatrix} 0 & \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} & U \\ 0_{4 \times 1} & \begin{bmatrix} A_{int} \end{bmatrix} & 0_{4 \times 1} \\ 0 & \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} & 0 \end{bmatrix}$$
(3.68)
$$B = \begin{bmatrix} 0 \\ \begin{bmatrix} B_{int} \\ 0 \end{bmatrix}$$
(3.69)

where A_{int} and B_{int} are the matrices of Eq. 3.65 and Eq. 3.66 respectively.

3.3.1 Roll Model with Terrain Input

Similar to the bicycle model derivation of Section 3.2.1, the equations of motion for the 3DOF roll model can also be modified to include terrain angle, ϕ_t . A diagram of the roll model on a banked slope is shown in Fig. 3.5.

For the 3DOF roll model with small angle approximations, the terrain affects the



Figure 3.5. Roll model on a banked slope.

equations of motion (Eq. 3.41, Eq. 3.43, and Eq. 3.45) in the following manner:

$$\Sigma F_y = F_f + F_r + mg\sin(\phi_t) = F_f + F_r + mg\phi_t \tag{3.70}$$

$$\Sigma M_x = -D_\phi - K_\phi \phi_r + m_s h_{sr} g(\phi_r + \phi_t)$$
(3.71)

$$\Sigma M_z = I_{zz} \dot{r} = aF_f - bF_r \tag{3.72}$$

Once again, the terrain angle does not affect the sum of moments about the vehicle's z-axis. These equations also assume a positive roll angle is measured clockwise from the roll center.

The terrain angle is treated as a linear input to the system dynamics, which can be written in the form

$$M_{int}\dot{\vec{x}} + N_{int}\vec{x} = F_{int} \begin{bmatrix} \delta_f \\ \phi_t \end{bmatrix}$$
(3.73)

where \vec{x} is the state vector defined in Eq. 3.60. The matrices M_{int} and N_{int} are identical to those derived in for the standard roll model and are expressed in Eq. 3.61 and Eq. 3.62 respectively. The new F_{int} matrix includes the effects of the terrain and is written as

$$F_{int} = \begin{bmatrix} -C_{\alpha f} & mg \\ 0 & m_s h_{sr}g \\ -aC_{\alpha f} & 0 \\ 0 & 0 \end{bmatrix}$$
(3.74)

The A and B matrices of the state space model are then obtained through the relationship found previously:

$$A = -M_{int}^{-1}N_{int} (3.75)$$

$$B = M_{int}^{-1} F_{int} \tag{3.76}$$

3.3.2 Roll Model with Tire Lag Dynamics

Similar to the bicycle model derivation of Section 3.2.2, tire lag effects for the roll model can be expressed as the first-order differential equations of Eq. 3.25 and Eq. 3.26. The front and rear tire forces are again added as states in the model.

Using Eq. 3.41, Eq. 3.43, Eq. 3.45, and Eq. 3.58, with the addition of Eq. 3.25 and Eq. 3.26, the M_{int} , N_{int} , and F_{int} matrices of

$$M_{int}\dot{\vec{x}} + N_{int}\vec{x} = F_{int}\delta_f \tag{3.77}$$

can be modified in the following form:

$$M_{int} = \begin{bmatrix} m & 0 & m_s h_{sr} & 0 & 0 & 0 \\ m_s h_{sr} & -I_{xz} & I_{xx} + m_s h_{sr}^2 & 0 & 0 & 0 \\ 0 & I_{zz} & -I_{xz} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
(3.78)
$$N_{int} = \begin{bmatrix} 0 & mU & 0 & 0 & -1 & -1 \\ 0 & m_s h_{sr}U & D_{\phi} & K_{\phi} - m_s h_{sr}g & 0 & 0 \\ 0 & 0 & 0 & 0 & -a & b \\ 0 & 0 & -1 & 0 & 0 & 0 \\ \frac{-C_{\alpha f}}{\sigma_f} & \frac{-aC_{\alpha f}}{\sigma_f} & 0 & 0 & \frac{U}{\sigma_f} & 0 \\ \frac{-C_{\alpha r}}{\sigma_r} & \frac{bC_{\alpha r}}{\sigma_r} & 0 & 0 & 0 & \frac{U}{\sigma_r} \end{bmatrix}$$
(3.79)
$$F_{int} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{-C_{\alpha f}U}{\sigma_f} \\ 0 \end{bmatrix}$$
(3.80)

When bank angle input, $\phi_t,$ is also considered as a second input, the F_{int} matrix is

written as

$$F_{int} = \begin{bmatrix} 0 & mg \\ 0 & m_s h_{sr}g \\ 0 & 0 \\ 0 & 0 \\ \frac{-C_{\alpha f}U}{\sigma_f} & 0 \\ 0 & 0 \end{bmatrix}$$
(3.81)

The state vector, \vec{x} , of this intermediate model is given as

$$\vec{x} = \begin{bmatrix} V \\ r \\ \dot{\phi}_r \\ \phi_r \\ \phi_r \\ F_f \\ F_r \end{bmatrix}$$
(3.82)

Finally, the A and B matrices of the state space model are obtained through the relationship found previously:

$$A = -M_{int}^{-1} N_{int} (3.83)$$

$$B = M_{int}^{-1} F_{int} \tag{3.84}$$

3.4 Comparison of Vehicle Models

This section aims to compare the dynamics of the vehicle models presented above. Specifically, four of the models were simulated: the 2DOF model with and without tire lag, and the 3DOF model with and without tire lag. The agreement between these models was studied at a low speed of 13.4 m/s (30 mph) and a high speed of 26.8 m/s (60 mph).

Symbol	Value	Unit
m	3255	kg
m_s	2956	kg
a	1.459	m
b	1.895	m
h	1.234	m
h_{sr}	0.781	m
$C_{lpha f}$	-120,000	N/rad
$C_{lpha r}$	-120,000	N/rad
T_r	1.615	m
I_{xx}	1830	kg-m ²
I_{yy}	6488	kg-m ²
I_{zz}	7913	kg-m ²
I_{xz}	500	kg-m ²
D_{ϕ}	4500	N-m-s/rad
K_{ϕ}	$145,\!330$	N-m/rad
σ_{f}	0.7	m
σ_r	0.23	m
g	9.81	m/s^2

Table 3.3. Vehicle parameters for 1989 GMC 2500 Pick-up Truck.

The simulations for each speed were also conducted for both a flat road and an 8 deg bank angle. An open-loop steering pulse was chosen as the input in order to visualize the subtle differences between each model. In particular, the lateral velocity and yaw rate were examined for each model due to their overall description of the vehicle motion.

Due to the focus of this thesis, vehicle parameters for a rollover-prone truck were used in the simulations (Table 3.3). The parameters shown in Table 3.3 were also used for all simulations and results obtained in subsequent chapters. The simulation results for the steering pulse on a flat road are presented in Fig. 3.6, while the results on an 8 deg bank angle are presented in Fig. 3.7.

It can be seen from both Fig. 3.6 and Fig. 3.7 that the models are quite similar, but do in fact exhibit small differences. The most evident difference is shown by the lateral velocity of the 2DOF model versus that of the 3DOF model. The included roll dynamics



Figure 3.6. Lateral velocity and yaw rate response of each vehicle model to an open-loop steering pulse at 13.4 m/s and 26.8 m/s on a flat road. Note: "TL" in the legend denotes the vehicle model with tire lag.

of the vehicle in the 3DOF model result in exaggerated lateral motion when compared to the rigid 2DOF model. This is especially evident in the low-speed simulation, where the roll dynamics of the vehicle have time to evolve given the sudden steering maneuver.

The influence of tire lag effects can also be seen from the lateral velocity and yaw rate response of the vehicle models. While the effect of tire lag is indeed minimal, there is noticeable lag in the vehicle response to changes in steering input. Once again, the differences are more apparent in the low-speed simulation. At higher speeds, the tires rotate faster and can respond to changes in steering more quickly; thus, the effect of tire lag becomes less prominent. This is evidenced by the close agreement between the models with and without tire lag in the high-speed simulation when compared to the low-speed simulation. Small, often ignored dynamics such as these can in fact influence vehicle behavior. Tire lag dynamics become especially important when considering situ-



Figure 3.7. Lateral velocity and yaw rate response of each vehicle model to an open-loop steering pulse at 13.4 m/s and 26.8 m/s on an 8 deg bank angle. Note: "TL" in the legend denotes the vehicle model with tire lag.

ations such as rollover, where delay in the tire response can amplify the vehicle's rollover

propensity.

Chapter 4 Vehicle Rollover Detection

4.1 Introduction

This chapter introduces the vehicle rollover metric applied in this thesis, the zero-moment point (ZMP) [40, 47, 48]. First, the concept of the zero-moment point is explained, followed by its derivation for the vehicle models discussed in Chapter 3. Next, the theory behind linear state preview is discussed and how this theory can be applied to predict a vehicle's roll behavior in the future. Finally, an analysis is performed to determine the minimum state preview time needed to prevent rollover under worst-case driving maneuvers.

4.2 Concept of Zero-Moment Point

This section discusses the concept behind the zero-moment point (ZMP) so that it can be understood intuitively. The zero-moment point is defined as the point on the ground where the summation of tipping moments, due to gravity and inertia forces, equals zero [49]. For a general object, if the ZMP moves outside the object's support polygon, i.e., the contact points of the object on the ground, it will overturn. This concept is



Figure 4.1. Free-body diagrams of a mass on a hinged surface.

illustrated in Fig. 4.1, where a mass is resting without slip on a hinged surface. In Fig. 4.1(a), the surface is perfectly level. This results in the reaction force acting directly beneath the object's center of mass in the middle of the object; this reaction point is the ZMP. As the surface is inclined in Fig. 4.1(b), this reaction shifts to the right in order to balance the object's weight and satisfy the definition of ZMP. Eventually, there exists an angle, as in Fig. 4.1(c), where the reaction force is no longer able to balance the object's weight (or inertial dynamics) and the ZMP moves outside the object's support polygon. This creates a tipping moment such that the mass is no longer stable and will overturn.

In the context of vehicle stability, the zero-moment point becomes very useful when applied as a vehicle rollover metric [40, 47, 48]. There are several advantages inherent in the zero-moment point that warrant its use in this research. First, the zero-moment point explicitly accounts for terrain effects in its derivation, as will be shown in Section 4.3. This is extremely valuable when evaluating the rollover propensity of a vehicle on realistic driving surfaces. Second, zero-moment point analysis does not require knowledge of any ground-vehicle forces. By treating the vehicle as a kinematic chain, the zero-moment point can be calculated through each body's net moment contribution. This allows the ZMP to be calculated through inertial measurements of the vehicle. Although the location of the zero-moment point exists in three-dimensional space, only its coordinate along the vehicle's body-fixed y-axis is of interest for the application to vehicle rollover [40]. This coordinate, termed y_{zmp} , is measured on the ground from the vehicle's centerline and is the metric used to evaluate the vehicle's rollover propensity. Now, as mentioned in the example of Fig. 4.1, if the ZMP moves outside the object's support polygon, the object will overturn. For a vehicle, the support polygon is defined by the points where the tires contact the ground (assumed to be at the center of the wheel hub). Thus, if y_{zmp} moves outside the tires $(|y_{zmp}| \geq \frac{T_r}{2})$, wheel lift will commence.

4.3 Formulation of Zero-Moment Point

4.3.1 Zero-Moment Point Derivation

This section derives the location of the ZMP for a vehicle. The derivation can be done for both the 2DOF bicycle model and the 3DOF roll model. Lapapong showed in previous work [40,47,48], however, that when applied to a real vehicle, the results of y_{zmp} for the bicycle model and roll model are nearly identical. Therefore, this paper only presents the y_{zmp} derivation for the simpler bicycle model. The reader is directed to [40] for full derivations of y_{zmp} for both models.

Consider the general kinematic chain shown in Fig. 4.2. In the illustration, the middle body is assumed to have the following properties: a mass of m_i , a translational velocity of \vec{v}_i , a translational acceleration of \vec{a}_i , an angular velocity of $\vec{\omega}_i$, an angular acceleration of $\vec{\omega}_i$, and an inertia tensor of I_i about its center of mass. Using the general equations of motion of the chain [54–56] and D'Alembert's principle [56], the sum of moments about point A can be written as

$$\vec{M}_A = \sum_i (\vec{p}_i \times m_i \vec{a}_i) + \sum_i (I_i \dot{\vec{\omega}}_i + \vec{\omega}_i \times I_i \vec{\omega}_i) - \sum_i (\vec{p}_i \times m_i \vec{g})$$
(4.1)



Figure 4.2. Generalized kinematic chain.

where $\vec{p}_i = \vec{r}_i - \vec{r}_{zmp}$. Point A then becomes the zero-moment point when $\vec{M}_A = \begin{bmatrix} 0 & 0 & M_z \end{bmatrix}^T$.

Now, the generalized kinematic chain of Fig. 4.2 can be applied to the 2DOF bicycle model, as seen in Fig. 4.3. The notation used throughout the derivation can be referred to back in Table 3.2. The location of the zero-moment point in three-dimensional space, with respect to the vehicle's body-fixed coordinate frame, is given by the vector

$$\vec{r}_{zmp} = x_{zmp}\hat{i} + y_{zmp}\hat{j} + z_{zmp}\hat{k}$$

$$\tag{4.2}$$

Recalling that the zero-moment point must lie on the ground, z_{zmp} can be expressed in terms of the terrain and vehicle properties shown in Fig. 4.3. When $\phi_r > \phi_t$, z_{zmp} is calculated as such:

$$z_{zmp} = h + \left(\frac{T_r}{2} - y_{zmp}\right) \tan(\phi_r - \phi_t)$$
(4.3)



Figure 4.3. Bicycle model on a banked slope with ZMP illustration.

In the case where $\phi_r < \phi_t$, Eq. 4.3 is slightly modified to produce the following:

$$z_{zmp} = h + \left(\frac{T_r}{2} + y_{zmp}\right) \tan(-\phi_r + \phi_t)$$
(4.4)

Combining Eq. 4.3 and Eq. 4.4 to account for all conditions results in the following:

$$z_{zmp} = h + \frac{T_r}{2} |\tan(\phi_r - \phi_t)| - y_{zmp} \tan(\phi_r - \phi_t)$$
(4.5)

Thus, Eq. 4.2 can be rewritten as such:

$$\vec{r}_{zmp} = x_{zmp}\hat{i} + y_{zmp}\hat{j} + \left[h + \frac{T_r}{2}|\tan(\phi_r - \phi_t)| - y_{zmp}\tan(\phi_r - \phi_t)\right]\hat{k}$$
(4.6)

Equation 4.6 effectively removes z_{zmp} from the vector describing the location of the zero-moment point and defines it in terms of x_{zmp} and y_{zmp} .

The remaining terms of Eq. 4.1 will now be discussed. The body-fixed coordinate system of the bicycle model is attached to the vehicle's center of mass; this means that $\vec{r}_i = 0$ and $\vec{p}_i = -\vec{r}_{zmp}$. Additionally, because the vehicle is not constrained in any direction, the velocities and accelerations of Eq. 4.1 can be expressed as the following:

$$\vec{\omega} = \dot{\phi}_r \hat{i} + \dot{\theta}\hat{j} + r\hat{k} \tag{4.7}$$

$$\dot{\vec{\omega}} = \ddot{\phi}_r \hat{i} + \ddot{\theta}\hat{j} + \dot{r}\hat{k} \tag{4.8}$$

$$\vec{a}_G = a_{Gx}\hat{i} + a_{Gy}\hat{j} + a_{Gz}\hat{k} \tag{4.9}$$

where \vec{a}_G is the linear acceleration of the center of mass. Due to the properties of the vehicle, the inertia tensor is defined as

$$I = \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}$$
(4.10)

where $I_{xy} = 0$ due to the assumption that the vehicle is symmetric about the xz-plane. Finally, the acceleration due to gravity of Eq. 4.1, when expressed in the vehicle's body-fixed coordinates, takes the form

$$\vec{g} = -g\sin(\theta)\hat{i} + g\sin(\phi_r)\cos(\theta)\hat{j} + g\cos(\phi_r)\cos(\theta)\hat{k}$$
(4.11)

Equations 4.6 - 4.11 can now be substituted into Eq. 4.1. Setting the x and y components equal to zero $(\vec{M}_A = \begin{bmatrix} 0 & 0 & M_z \end{bmatrix}^T)$ and rearranging produces the nonlinear solution for y_{zmp} :

$$y_{zmp} = \{ mg\cos(\theta)\sin(\phi_r)[T_r|\tan(\phi_r - \phi_t)| + 2h] - ma_{Gy}[T_r|\tan(\phi_r - \phi_t)| + 2h] - 2I_{xx}\ddot{\phi}_r + 2I_{xz}\dot{r} + 2I_{yz}(\dot{\theta}^2 - r^2) + 2(I_{xz} + I_{yy} - I_{zz})\dot{\theta}r \}$$

$$/\{2m[g\cos(\theta)\cos(\phi_t)\sec(\phi_r - \phi_t) - a_{Gy}\tan(\phi_r - \phi_t) - a_{Gz}] \}$$

$$(4.12)$$

A linearized version of y_{zmp} , found in [48], is expressed as the following:

$$y_{zmp} = -\frac{I_{xx}}{mg}(\ddot{\phi}_r + \ddot{\phi}_t) + h_{sr}(\phi_r + \phi_t) - \frac{h_{sr}}{g}a_{Gy}$$
(4.13)

Equation 4.13 is the formulation of y_{zmp} that is used for the remainder of the paper.

4.3.2 Inclusion of y_{zmp} in the Vehicle Model

The reader should now notice that Eq. 4.13 is essentially a linear combination of the vehicle's states and inputs for the roll model, where, as discussed in Chapter 3, the bank angle of the road is treated as a linear input to the system. While $\ddot{\phi}_r$ and a_y are not states themselves, they can be expressed as linear combinations of the states in the form

$$\ddot{\phi}_r = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} A\vec{x} + B\vec{u} \end{bmatrix}$$
(4.14)

$$a_y = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A\vec{x} + B\vec{u} \end{bmatrix} + Ur$$
(4.15)

where the state vector of interest for the roll model is

$$\vec{x} = \begin{bmatrix} y \\ V \\ r \\ \dot{\phi}_r \\ \phi_r \\ \phi_r \\ \psi \end{bmatrix}$$
(4.16)

and the input vector is

$$\vec{u} = \begin{bmatrix} \delta_f \\ \phi_t \end{bmatrix} \tag{4.17}$$

Substituting Eq. 4.14 and Eq. 4.15 into Eq. 4.13 and rearranging terms produces y_{zmp} as an explicit linear combination of states and inputs

$$y_{zmp} = \begin{bmatrix} 0 & -\frac{h_{sr}}{g} & 0 & -\frac{I_{xx}}{mg} & 0 & 0 \end{bmatrix} [A\vec{x} + B\vec{u}] + h_{sr} \left(\phi_r + \phi_t - \frac{Ur}{g}\right)$$
(4.18)

where $\ddot{\phi}_t = 0$ by assuming that the terrain bank angle is constant. Thus, Eq. 4.18 can be used to add y_{zmp} as an output of the system.

4.4 Formulation of Previewed Zero-Moment Point

4.4.1 Generalized Preview Solution of a Linear Dynamic System

The methodology for obtaining previewed state information will now be introduced. Predicting the states of a dynamic system is done by extending the current states and inputs over a specified preview horizon, i.e., the time interval into the future. Considering the following linear system,

$$\dot{\vec{x}}(t) = A\vec{x}(t) + B\vec{u}(t)$$

$$\vec{y}(t) = C\vec{x}(t) + D\vec{u}(t)$$

$$(4.19)$$

the general solution of the state vector at the preview time is given by the following:

$$\vec{x}(t+T) = \Phi(t+T,t)\vec{x}(t) + \int_{t}^{t+T} \Phi(t+T,\tau)B(\tau)u(\tau)d\tau$$
(4.20)

where T is the preview interval and Φ is the system's state transition matrix. The state transition matrix is determined by the Peano-Baker series, which for a linear timeinvariant system, reduces to the matrix exponential in the form

$$\Phi(t+T,t) = \sum_{k=0}^{\infty} \frac{A^k T^k}{k!} = e^{AT}$$
(4.21)

The solution given by Eq. 4.20 and Eq. 4.21 produces the state vector over the preview horizon given the current state vector and the input over the preview horizon. This solution can be simplified by assuming that the system is time-invariant and that the inputs remain constant over the (short) preview interval. Additionally, further simplification is possible for the case in which only the state vector at time t + T is desired, known as single point preview, rather than over the entire preview horizon. These assumptions reduce Eq. 4.20 to the following:

$$\vec{x}(t+T) = \Phi(t+T,t)\vec{x}(t) + \Psi T B \vec{u}(t)$$

$$(4.22)$$

where Ψ is defined as

$$\Psi = I + \sum_{k=1}^{\infty} \frac{(TA)^k}{(k+1)!}$$
(4.23)

and I is the identity matrix. It should be noted that Eq. 4.23 does not quickly converge when the elements of the A matrix are large. Therefore, an alternate calculation is presented in Section 4.5.3 for these cases.

Equation 4.22 and the state transition matrix definition of Eq. 4.21 can be applied to obtain the final previewed state vector. For this paper, the terms A_p and B_p are used to identify the state and input matrices, respectively, of the previewed state vector. These take the form

$$A_p = \Phi(t+T,t) = e^{AT} \tag{4.24}$$

$$B_p = \Psi T B \tag{4.25}$$

such that Eq. 4.22 is written as

$$\vec{x}(t+T) = A_p \vec{x}(t) + B_p \vec{u}(t)$$
 (4.26)

4.4.2 Inclusion of Previewed y_{ZMP} in the Vehicle Model

The solution of the previewed state vector can now be applied in the context of vehicle rollover. For this application, the desired output of the system is the previewed value of y_{zmp} . This information allows decisions regarding vehicle control to be made based on the vehicle's predicted roll stability in the future. The previewed value of y_{zmp} , termed $y_{zmp}(t+T)$, can be obtained by selecting the appropriate C and D output matrices of the state space model.

Equation 4.13 must now be formulated as a linear combination of the previewed vehicle states. In a similar fashion to that of Section 4.3.2, the roll acceleration at the preview time is given by

$$\ddot{\phi}_r(t+T) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} A\vec{x}(t+T) + B\vec{u}(t+T) \end{bmatrix}$$
(4.27)

Recognizing that all inputs are assumed to be constant over the preview horizon such that $\vec{u}(t+T) = \vec{u}(t)$ and substituting Eq. 4.26 gives the future roll acceleration based on the current states and inputs:

$$\ddot{\phi}_r(t+T) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} A[A_p \vec{x}(t) + B_p \vec{u}(t)] + B \vec{u}(t) \end{bmatrix}$$
(4.28)

Similarly, the previewed roll angle and previewed lateral acceleration can be written in this manner as

$$\phi_r(t+T) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_p \vec{x}(t) + B_p \vec{u}(t) \end{bmatrix}$$
(4.29)
$$a_{y}(t+T) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A[A_{p}\vec{x}(t) + B_{p}\vec{u}(t)] + B\vec{u}(t)] \\ + U\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} A_{p}\vec{x}(t) + B_{p}\vec{u}(t) \end{bmatrix}$$
(4.30)

Substituting Eq. 4.28 - 4.30 into Eq. 4.13 gives y_{zmp} at the preview time as a function of the current states and inputs:

$$y_{zmp}(t+T) = \begin{bmatrix} 0 & -\frac{h_{sr}}{g} & 0 & -\frac{I_{xx}}{mg} & 0 & 0 \end{bmatrix} \begin{bmatrix} A[A_p\vec{x}(t) + B_p\vec{u}(t)] + B\vec{u}(t)] \\ + \begin{bmatrix} 0 & 0 & -\frac{h_{sr}U}{g} & 0 & h_{sr} & 0 \end{bmatrix} \begin{bmatrix} A_p\vec{x}(t) + B_p\vec{u}(t)] + h_{sr}\phi_t \end{bmatrix}$$
(4.31)

Once again, Eq. 4.31 can be used to add previewed y_{zmp} as an output of the state space model.

4.5 Minimum Preview Time Needed to Prevent Vehicle Rollover

An important application of previewed y_{zmp} is to detect when a vehicle's present steering behavior, along with its current state, could soon result in wheel lift. Identifying impending rollover in the future allows the vehicle to display warnings to the driver or execute a corrective intervention in order to mitigate the risk. For actions such as these to be feasible, however, it is necessary to determine the minimum preview time necessary to predict and prevent wheel lift. One of the key tuning parameters in predictive control approaches is the preview horizon length. Insufficient preview does not provide enough advanced warning, whereas too much preview degrades the accuracy of the predictive state calculation. Additionally, the length of the preview horizon greatly affects the computation time for MPC approaches; this means it is desirable to keep the preview horizon as short as possible. However, in much of the literature discussed in Chapter 2, the preview horizon with respect to rollover is often determined using a guess/check methodology. The literature [17, 18, 24–28, 45] suggests a range of preview horizons between 0.2 sec and 2.0 sec, which is a wide window of uncertainty. Therefore, the goal of this section is to explicitly determine the necessary preview time for representative rollover prevention strategies.

To this end, a "worst-case" driving situation for rollover, as defined in [48], was considered such that the vehicle has a high rollover propensity. The vehicles with the highest risk of rollover are typically SUVs and trucks; therefore, the parameters for a 1989 GMC 2500 pick-up truck were used due to the vehicle's high center-of-gravity and its availability for testing. The vehicle parameter values used in the simulations, as well as throughout the rest of the paper, can be seen in Table 3.3. In addition to rolloverprone parameters, the vehicle was assumed to be travelling at 26.8 m/s (60 mph) and experiencing a terrain bank angle of 8 deg (14% superelevation), a value typical of the road profile on a sharp highway curve [57].

Under these circumstances, simulations were performed in which the driver steers up the slope of the road, meant to approximate a driver performing an evasive maneuver with a severe course change. Specifically, the initial driver steering input, δ_f , follows a sinusoidal trajectory to a desired steering magnitude and then remains constant in the following form:

$$\delta_f = \begin{cases} \frac{A}{2} \sin\left(2\pi ft - \frac{\pi}{2}\right) + \frac{A}{2} & \text{for} \quad 0 < t \le \frac{1}{2f} \\ A & \text{for} \quad t > \frac{1}{2f} \end{cases}$$
(4.32)

where A is the steering angle magnitude (rad) of the tires, and f is the steering frequency (Hz). A scenario such as this would occur if a driver is attempting to avoid an obstacle on the road, or if he/she has drifted off the road and is trying to correct his/her course.

4.5.1 Corrective Steering Maneuver #1

In order to mitigate the rollover threat in the scenario presented above, a corrective steering action was chosen as the intervention strategy if the vehicle predicted imminent wheel lift, i.e., $|y_{zmp}(t+T)| \geq \frac{T_r}{2}$. The corrective action considered in this section is open-loop in nature. An open-loop analysis is beneficial to examine and understand the preview horizon because it limits the coupling between a feedback algorithm and the behavior of y_{zmp} . The simplicity of an open-loop framework also allows for a thorough development of the preview horizon's effect on roll stability, without narrowing the intervention to a particular steering law. This way, the results that follow can be modified for other control inputs, such as braking, or extended for use with closed-loop intervention strategies, as will be discussed in Chapter 6. It should also be noted that the following open-loop steering actions are not considered optimal intervention strategies and are not suggested to be implemented in practice. Rather, these maneuvers are simply meant to represent the gross behavior of a hypothetical driver in an emergency scenario and subsequently examine their effect on the preview horizon necessary to prevent wheel lift.

The corrective steering intervention was implemented such that if the vehicle detects imminent wheel lift, the steering input follows a sinusoidal trajectory (of the same frequency as Eq. 4.32) back to a zero steering input in the form:

if
$$y_{zmp}(t+T) \ge \frac{T_r}{2}$$
 at $t = t^*$

$$\delta_f = \begin{cases} \frac{A}{2} \sin\left(2\pi f t + \frac{\pi}{2}\right) + \frac{A}{2} & \text{for } t^* < t \le t^* + \frac{1}{2f} \\ 0 & \text{for } t > t^* + \frac{1}{2f} \end{cases}$$
(4.33)

A generalized plot of Eq. 4.32 with the corrective steering intervention of Eq. 4.33 is shown in Fig. 4.4 for further understanding. The axes of Fig. 4.4 have purposefully been generalized for any value of A and f.



Figure 4.4. Generalied example plot of the steering input for corrective steering maneuver #1 simulations.

Simulations of the corrective steering intervention shown in Fig. 4.4 investigated how the severity of the initial driver steering input, and subsequent corrective action, affect the minimum preview time needed to prevent wheel lift. This consisted of testing combinations of steering angle (over the range allowed by the limits of the steering rack) and steering frequency (over the range of feasible maneuvers). It was also assumed that the tires did not skid for the simulated maneuvers. Lapapong showed in [48] that wheel lift will occur before the tires skid for steering frequencies below 0.9 Hz. Thus, results for steering combinations above this frequency should be considered questionable, as skidding is likely to precede rollover.

An iterative simulation structure was used to determine the minimum preview time needed to prevent wheel lift. A successful rollover prevention was defined to have occurred when the current ("real-time") value of y_{zmp} remained within the vehicle's track width for the entirety of the maneuver. Thus, the wheel lift thresholds for the vehicle are defined as $-\frac{T_r}{2} \leq y_{zmp} \leq \frac{T_r}{2}$, since y_{zmp} is measured from the vehicle's centerline.



Figure 4.5. Example of the iterative preview time calculation for corrective steering maneuver #1 simulations. This plot was created for a steering combination of -8.5 deg steering angle and 0.55 Hz steering frequency, resulting in a necessary preview time of 0.33 s.

For each steering combination, the preview time was iteratively increased from zero by 0.01 sec until the corrective steering maneuver succeeded in preventing wheel lift. An example of this process is shown in Fig. 4.5, where y_{zmp} has been normalized by dividing by half the track width such that values above 1 or below -1 signify wheel lift. This figure shows how the peak y_{zmp} value decreases as the preview time is increased, up until the point where the vehicle remains beneath the wheel lift threshold. To asses the fidelity of the linear model, the simulations were also performed using the nonlinear simulation package CarSim. In CarSim, the preview time was iteratively increased in the same fashion until the vertical force on every tire remained positive (no wheel lift) for the entirety of the maneuver.

Figure 4.6 shows the results of one specific steering combination, simulated using the minimum preview time for that combination. Specifically, Fig. 4.6(a) illustrates how the intervention strategy of Eq. 4.33, implemented when $y_{zmp}(t+T)$ rises above the wheel lift threshold, is able to keep the current value of y_{zmp} below this threshold. When the



Figure 4.6. y_{zmp} (a) with and (b) without corrective steering maneuver #1. This plot was created for a steering combination of -8.5 deg steering angle and 0.55 Hz steering frequency with a preview time of 0.33 s.

same initial driver input is simulated without the intervention strategy, shown in Fig. 4.6(b), the vehicle clearly passes the threshold and experiences wheel lift. Therefore, these results show that $y_{zmp}(t+T)$ can be used as an indicator to prevent future wheel lift. Figure 4.6 also shows that the roll model and CarSim results agree quite well, with the roll model behaving slightly more conservative as indicated by the higher peak values. This suggests that the linear roll model can be used in further study.

The simulations of Fig. 4.5 and Fig. 4.6 were then repeated over the entire test suite of steering combinations, using the same methods described above to determine the necessary preview time. The results of this analysis are shown in Fig. 4.7 in the form of a contour plot for the linear roll model (top) and CarSim (bottom). This plot indicates that longer preview times are needed for low frequency maneuvers (lower frequency means a less severe steering input). This relationship can be explained from the Bode plots between steering input - y_{zmp} and steering input - $y_{zmp}(t + T)$, shown in Fig. 4.8. The frequency response of y_{zmp} exhibits a notch filter effect at higher frequencies, while the frequency response of $y_{zmp}(t + T)$ remains relatively constant. Thus, because the $y_{zmp}(t + T)$ calculation assumes a constant steering input over the preview horizon,



Figure 4.7. Contour plots of the minimum preview times (sec) needed to prevent wheel lift for corrective steering maneuver #1 for (top) linear roll model and (bottom) CarSim.

steering inputs at higher frequencies cause $y_{zmp}(t+T)$ to rise quickly when compared to y_{zmp} . This is a result of the high gain difference between $y_{zmp}(t+T)$ and y_{zmp} and leads to earlier detection and correction of the rollover threat. Impending wheel lift from low frequency steering inputs, however, requires more preview to detect a threat due to the low gain difference, where $y_{zmp}(t+T)$ and y_{zmp} rise with approximately the same slope.

Overall, the longest preview time of the steering combinations tested for corrective maneuver #1 was 0.66 sec for a 23 deg, 0.16 Hz steering input.

4.5.2 Corrective Steering Maneuver #2

A second, slightly more severe, corrective steering intervention was also simulated that consists of two separate corrections: one steering away from the direction of travel, and one correcting back toward the direction of travel. This maneuver was designed



Figure 4.8. Bode plot of steering input - y_{zmp} and steering input - $y_{zmp}(t+T)$.

to more closely resemble the NHTSA Fishhook prescribed for rollover testing described in Chapter 2 [34]. A second consideration of this maneuver is that it keeps the vehicle traveling in its original direction, but with an offset, as opposed to corrective steering maneuver #1 which steers the vehicle away. To remain in the original direction of travel, the vehicle must steer back in similar fashion to that of a lane change. This could potentially result in an overcorrection that puts the vehicle at risk. Thus, the scenario considered in this section consists of a course departure, followed by a course correction.

All conditions and parameters remain the same for this new maneuver, however, now if the vehicle detects impending wheel lift during the initial steering input, it implements a sinusoidal correction to the opposite steering angle (overcorrection). Additionally, because this overcorrection is potentially dangerous itself, if the vehicle detects impending



Figure 4.9. Generalized example plot of the steering input for corrective steering maneuver #2 simulations.

wheel lift during the overcorrection, it then returns to a zero steering input. This intervention strategy is illustrated graphically in Fig. 4.9 with generalized axes and described mathematically by the following:

$$\text{if } y_{zmp}(t+T) \ge \frac{T_r}{2} \text{ at } t = t^* \\ \delta_f = \begin{cases} A \sin\left(2\pi ft + \frac{\pi}{2}\right) & \text{for } t^* < t \le t^* + \frac{1}{2f} \\ -A & \text{for } t > t^* + \frac{1}{2f} \end{cases}$$

$$\text{if } y_{zmp}(t+T) \le \frac{-T_r}{2} \text{ at } t = t^{**} \\ \delta_f = \begin{cases} -\frac{A}{2} \sin\left(2\pi ft + \frac{\pi}{2}\right) - \frac{A}{2} & \text{for } t^{**} < t \le t^{**} + \frac{1}{2f} \\ 0 & \text{for } t > t^{**} + \frac{1}{2f} \end{cases}$$

$$(4.34)$$

This open-loop steering trajectory, specified by Eq. 4.32 and Eq. 4.34, now mimics a driver that is aggressively steering to both avoid an obstacle and remain in the original direction of travel. Once again, this framework is not meant to produce an optimal



Figure 4.10. Example of the iterative preview time calculation for corrective steering maneuver #2 simulations. This plot was created for a steering combination of -8.5 deg steering angle and 0.55 Hz steering frequency, resulting in a necessary preview time of 0.3 s.

control law, but instead to create repeatable driver behavior used to analyze the necessary preview horizon.

The iterative procedure for determining the necessary preview time remains the same for this new steering intervention and is illustrated in Fig. 4.10. It can be seen in Fig. 4.10 that sufficient preview is now needed to prevent wheel lift on both sides of the vehicle due to the overcorrection. The simulation outputs for the roll model and CarSim are shown in Fig. 4.11 for one specific steering combination. Figure 4.11(a) shows that corrective steering maneuver #2 is able to prevent wheel lift for both the initial driver steering input and the subsequent overcorrection, while 4.11(b) indicates wheel lift has occurred for the same maneuver without secondary intervention.

Once again, the simulation procedures were repeated over the test suite of steering combinations to determine the necessary preview time for corrective steering maneuver #2. The results are shown in Fig. 4.12 in the form of a contour plot. Overall, the necessary preview times exhibit the same trends as those seen in Section 4.5.1; however,



Figure 4.11. y_{zmp} (a) with and (b) without corrective steering maneuver #2. This plot was created for a steering combination of -8.5 deg steering angle and 0.55 Hz steering frequency with a preview time of 0.3 s.

slightly less preview (approximately 0.1 sec) is needed for roll model predictions using corrective steering maneuver #2. This is due to the fact that the initial driver steering input is still the main threat and the overcorrection addresses this threat more aggressively. The similarity of the results between the two intervention strategies suggests that further analysis of third, fourth, etc. corrections is probably not necessary.

4.5.3 Tire Lag Effects on Preview Horizon

As noted in Chapter 3, tire lag effects are important to consider when analyzing the roll dynamics of a vehicle. The delay of the tire response could ultimately amplify the vehicle's rollover propensity under certain driving conditions. This is especially true for steering maneuvers, as the vehicle cannot immediately generate lateral force. The influence of tire lag dynamics is also dependent on the longitudinal velocity of the vehicle. As the tires rotate faster, they are able to generate lateral force and respond to changes in steering more quickly. Thus, tire lag effects become more prominent at lower speeds.

To address these concerns, simulations of corrective steering maneuver #1 were performed inclusive of tire lag dynamics. Corrective steering #2 simulations were not re-



Figure 4.12. Contour plots of the minimum preview times (sec) needed to prevent wheel lift for corrective steering maneuver #2 for (top) linear roll model and (bottom) CarSim.

peated due to the similarity of results and shorter preview horizons. Tire relaxation values of 0.7 m and 0.23 m were used for the front and rear tires, respectively. These values were found by obtaining a range of standard values from [58] that were then tuned for the test vehicle.

Initial simulations showed that the infinite series of Eq. 4.25 and Eq. 4.23 used to calculate B_p converged too slowly to be applied to the roll model inclusive of tire lag. This resulted from the fact that several terms of the tire lag model state space matrices were orders of magnitude larger than terms of the state space matrices without tire lag. Further testing indicated that this also occurred for long preview times (greater than 1.0 sec), even in the standard roll model. To address this problem, an alternate method [59] was used to calculate B_p in discretized form using small time steps, τ , added up over the preview horizon in the following form:

$$B_p = \sum_{m=0}^{n-1} (A_p^{n-1-m}) B\tau$$
(4.35)

$$B_p = \sum_{m=0}^{n-1} [e^{A(T-t)(n-1-m)}] B\tau$$
(4.36)

where $n = T/\tau$. If the time step size is small enough, the Euler approximation of A_p can be used instead such that Eq. 4.35 becomes

$$B_p = \sum_{m=0}^{n-1} [(I + A\tau)^{n-1-m}] B\tau$$
(4.37)

Equation 4.35 provides a more robust solution of the previewed input matrix, B_p . One drawback of this approach, however, is that it is computationally expensive, hence why it was not used in previous simulations.

Using the new formulation of B_p , the simulations were repeated over the test suite of steering combinations with the tire lag model. To examine the effect of longitudinal speed, the simulations were also performed over a range of speeds from 11.2 m/s (25 mph) to 26.8 m/s (60 mph). Once again, the vehicle parameters of Table 3.3 were used with a bank angle of 8 deg (14 % superelevation). Figure 4.13 shows the results of the simulations with and without tire lag for the steering combination that required the most preview. Plots of the remaining steering combinations are not included; however, the results exhibit the same trends as those seen in Fig. 4.13. Overall, the effect of tire lag on the necessary preview time is relatively small. Speeds below 16 m/s saw an increase of the preview time of approximately 0.04 s while speeds above 16 m/s saw an increase of only 0.01 - 0.02 s. A second insight is that the relationship between preview time and speed appears to be approximately linear; however, further testing should be done to confirm this relationship.



Figure 4.13. Preview times needed to prevent wheel lift for varying longitudinal velocities (-23 deg, 0.2 Hz steering combination)

4.5.4 Preview Horizon for Modified Vehicle Configurations

So far, Section 4.5 has only considered a worst-case vehicle configuration for rollover. Most vehicles, however, are not subject to these conditions and have a lower rollover propensity; therefore, they should not need as much preview to mitigate the risk. To test this hypothesis, corrective steering maneuver #1 simulations (without tire lag at 26.8 m/s and with an 8 deg bank angle) were once again repeated, but for vehicle configurations with an artificially decreased CG height. Two variations of the truck used in the simulations above were tested, with parameter modifications shown in Table 4.1. In an effort to generalize the minimum preview curves, each vehicle variation was also correlated with its corresponding Static Stability Factor (SSF) [35], where the SSF was defined previously as

$$SSF = \frac{T_r}{2h} \tag{4.38}$$

Vehicle Variation	h	h_{sr}	SSF
Original $(#1)$	$1.234 { m m}$	$0.781~\mathrm{m}$	0.654
#2	1.0 m	$1.06 \mathrm{~m}$	0.808
#3	0.7 m	$0.75 \mathrm{~m}$	1.155

Table 4.1. Variations to the CG height of the simulated 1989 GMC 2500 Pick-up Truck.

The SSF is used here because it offers a generalized rating for rollover resistance based purely on the vehicle's physical design, allowing one to obtain information about the necessary preview horizon in the absence of a dynamic analysis. Table 4.1 also includes the SSF value for each vehicle variation.

The results of the simulations for the modified truck parameters are shown in Fig. 4.14. While the same trends of Fig. 4.7 and Fig. 4.12 are present, it can be seen that less



Figure 4.14. Contour plots of the minimum preview times (sec) needed to prevent wheel lift for corrective steering maneuver #1 for (top) vehicle variation #2 and (bottom) vehicle variation #3.



Figure 4.15. Preview times needed to prevent wheel lift for corrective steering #1 for varying vehicle configurations (-23 deg, 0.2 Hz steering combination). SSF is the vehicle's Static Stability Factor, a measure of rollover propensity.

preview is needed for wheel lift prevention as the CG height is decreased. As expected, Fig. 4.14 also shows an increase in the "safe zone" where no rollover occurs. Specifically, lower steering magnitudes do not induce wheel lift as the CG height is decreased.

Figure 4.15 shows how preview horizon information can be generalized by correlating each vehicle variation's SSF value with the necessary preview time. The results are shown once again for the steering combination that required the most preview. Plots of the remaining steering combinations are not included; however, the results exhibit the same trends as those seen in Fig. 4.15. Ultimately, generalized information such as this could be used as a guide for quickly choosing an estimate of the required preview horizon when designing intervention strategies.



Figure 4.16. Contour plot of minimum preview times (sec) needed to prevent wheel lift for corrective steering #1 for (top) a flat road (0-deg bank angle) and (bottom) an intermediate slope (4-deg bank angle).

4.5.5 Bank Angle Effects on Preview Horizon

Finally, this section examines how the preview horizon changes for different road superelevation values. The bank angle used in the simulations of the previous sections (8 deg) was meant to create a "worst-case" situation for rollover. While this value is representative of the terrain seen on a sharp highway curve [57], it is relatively steep compared to the majority of driving conditions. Therefore, the corrective steering maneuver #1simulations were repeated for two additional bank angle values: a flat road (0 deg) and an intermediate slope (4 deg) using the vehicle parameters of Table 3.3.

Figure 4.16 shows minimum preview horizons necessary for the different bank angles in the form of a contour plot. The results of Fig. 4.16 are intuitive and follow the trends seen in the previous sections. The milder bank angles decrease the rollover propensity of the vehicle, resulting in slightly smaller minimum preview horizons. Additionally, an increase in the "safe zone" where no rollover occurs is seen for the flat road simulations. Overall, the ranges of preview times for the 0-deg slope (0.07 sec - 0.55 sec) and 4-deg slope (0.1 sec - 0.55 sec) are similar to the range seen in Section 4.5.1 for the 8-deg slope. On average, the minimum preview horizons decreased by 0.06 sec for both the 0-deg slope and 4-deg slope when compared to the 8-deg slope.

Chapter 5 Open-Loop Collision Avoidance and Rollover Prevention

5.1 Introduction

This chapter focuses on rollover prevention in the context of collision avoidance; specifically, the aim is to determine the minimum intervention distance needed for a vehicle to avoid a collision, while also considering factors such as wheel lift and yaw stability. An open-loop steering analysis is used to investigate varying degrees of lane change avoidance trajectories. The vehicle's roll stability and handling stability are then analyzed for each open-loop lane change maneuver.

The collision avoidance scenario considered in this work is presented in Fig. 5.1. As shown in the diagram, the scenario considers a vehicle traveling at constant velocity that is approaching a stationary object. A last-minute avoidance maneuver in the form of a lane change must be executed in order to avoid this obstacle. This situation is not uncommon; a scenario such as this could occur for an inattentive driver that is approaching a traffic jam or a pedestrian in the street. The diagram also shows that as



Figure 5.1. Collision avoidance scenario: a moving vehicle approaching a stationary object.

the vehicle approaches the object, the avoidance maneuver becomes progressively more severe, with the last possible intervention distance illustrated by D_2 . While the maneuver at distance D_2 is capable of avoiding the obstacle, the severity may result in wheel lift or loss of yaw stability, as indicated by tire skidding. Therefore, the vehicle may need to intervene earlier, for example at distance D_1 or D_0 . Thus, the purpose of this chapter is to determine how the threat of wheel lift affects the minimum intervention distance needed for a safe avoidance maneuver.

First, an idealized scenario is reviewed to determine the minimum intervention distance when wheel lift and tire skid are not considered as factors in the avoidance strategy. Next, the methodology for determining the open-loop lane change maneuvers is discussed. Third, the steering intervention strategy for collision avoidance with wheel lift and tire skid prevention is explained. Finally, the results of the analysis are presented showing the minimum intervention distance needed for both flat road and banked curve simulations.

5.2 Collision Avoidance Without Rollover Consideration

In order to determine how wheel lift and tire skid consideration affect the minimum intervention distance, it is first necessary to provide a baseline for the idealized intervention distance. The scenario of a moving vehicle approaching a stationary object was considered as part of the work done by Jansson [20]. In this work, Jansson considers both a pure braking intervention and a pure steering intervention. The minimum intervention distances for each intervention strategy used in [20] are derived using constant acceleration equations and given as

$$D_{min,brake} = \frac{U_0^2}{2a_x} \tag{5.1}$$

$$D_{min,steer} = \sqrt{\left(\frac{U^2}{2a_y}\right)(T_{r,host} + T_{r,obj}) + \frac{(T_{r,host})^2 - (T_{r,obj})^2}{4}}$$
(5.2)

where a_x and a_y are the longitudinal and lateral acceleration, respectively. The width of the vehicle and object are assumed to be 2 m and the vehicle is assumed to achieve instantaneous accelerations of 9.82 m/s² for both steering and braking.

The minimum intervention distance for both intervention strategies as a function of speed as derived in [20] are shown in Fig. 5.2. Here we can see that for low speeds, braking provides a more effective intervention strategy, whereas at speeds greater than roughly 12.5 m/s (28 mph), pure steering required less intervention distance. Thus, because this work does not consider braking intervention, only high speeds (above 12.5 m/s) are considered hereafter.

The results found in [20] provide a good baseline for the idealized distance needed to avoid a collision, however, they are also limited in several factors. As previously stated, these results assume the vehicle is able to achieve instantaneous acceleration in



Figure 5.2. Minimum intervention distance to avoid a collision with a stationary object for pure braking and pure steering, as found in [20].

the longitudinal and lateral directions, an assumption which neglects the dynamics of the vehicle. Additionally, these results do not consider factors such as wheel lift, tire skid, or keeping the vehicle on the road during the avoidance maneuver. The purpose of the remainder of the chapter is not to suggest an optimized avoidance trajectory, but to consider these factors in determining the minimum intervention distance for collision avoidance.

5.3 Determination of Open-Loop Lane Change Steering Inputs

The avoidance maneuver considered in this chapter is an open-loop lane change, as illustrated in Fig. 5.1. The idea is that the vehicle "knows" that its avoidance trajectory will take this form under the appropriate circumstances. However, the severity of this maneuver depends on how fast the vehicle is traveling and at what distance to the obstacle the lane change is performed. This trajectory is given by

$$y = \frac{w_L}{2} \cos\left(\frac{\pi U}{D}t\right) - \frac{w_L}{2} \tag{5.3}$$

where y is the lateral position of the vehicle, D is the distance to the obstacle, and w_L is the width of the lane, here assumed to be the standard width of 3.65 m (12 ft).

In order for an open-loop maneuver to be successful, the vehicle must be able to execute a steering command that satisfies Eq. 5.3 given its current speed and distance to the obstacle. This open-loop steering command takes the form

$$\delta_f = A\sin(\omega t) \tag{5.4}$$

where A is the steering amplitude and ω is the steering frequency. Essentially, this consists of a lookup table of steering combinations that result in the lane change trajectory of Eq. 5.3. This strategy only requires that the vehicle decide the proper steering magnitude and frequency for the open-loop maneuver.

For this work, an iterative algorithm was used to determine the correlation between Eq. 5.3 and Eq. 5.4. Lane change simulations were performed on the 2DOF vehicle model over a fixed set of steering magnitudes, while the steering frequency was held constant according to

$$\omega_{steer} = 2\omega_{lane} = \frac{2\pi U}{D} \tag{5.5}$$

The lateral deviation of the vehicle over each lane change simulation was recorded, and the steering combination that resulted in the correct 3.65 m deviation was interpolated from the data. This process was then repeated over all values of U and D. Vehicle parameters for a 1989 GMC 2500 pick-up truck, provided in Table 3.3, were used once again due to the vehicle's high center of gravity. The results of this iterative analysis are



Figure 5.3. Results of the iterative analysis to determine the steering magnitudes and frequencies that result in an open-loop 3.65 m (12 ft) lane change.

shown in Fig. 5.3.

Equation 5.5 results from the fact that the lane change must be completed when the vehicle reaches the obstacle. This requirement ensures the vehicle clears the obstacle regardless of vehicle or obstacle size (assuming the obstacle does not penetrate the opposite lane) and also provides a conservative estimate of the minimum intervention distance.

5.4 Steering Intervention for Collision Avoidance on a Flat Road

Once again, an open-loop steering approach was chosen for the preliminary analysis because it has several benefits when considering last-minute avoidance maneuvers. The biggest advantages of open-loop control are its simplicity and speed. Open-loop control does not require the vehicle to calculate an optimized trajectory at every time step, as is the case for MPC approaches. An open-loop framework also does not need to wait for the growth of an error term to react, making it faster than a closed-loop implementation in an emergency scenario. The vehicle only needs to detect the obstacle (and any in its surroundings) and perform the intervention. This framework allows the operator to directly apply any control action and analyze its effects. Therefore, the results of the analysis performed here can be used in closed-loop design, as will be done in Chapter 6.

5.4.1 Simulation Overview

The open-loop intervention strategy will now be discussed. As previously stated, the purpose of the analysis is to determine the minimum intervention distance needed for a safe avoidance maneuver. Here, an avoidance maneuver is deemed safe if the vehicle avoids the obstacle, remains on the road, prevents wheel lift, and prevents tire skid. The criteria of avoiding the obstacle and staying on the road, however, have already been



Figure 5.4. Steering intervention decision flowchart for open-loop collision avoidance with rollover and tire skid consideration.

satisfied in the open-loop mapping from Eq. 5.3 to Eq. 5.4. Thus, simulations of the lane change maneuvers found in Section 5.3 were performed on a flat road with the roll model to analyze the rollover and tire skid criteria.

The decision flowchart of Fig. 5.4 outlines this process. As the vehicle approaches the obstacle, it continually plans its avoidance trajectory according to Eq. 5.3 and looks up the appropriate steering according to Eq. 5.4. It then calculates the maximum value of y_{zmp} and $\alpha_{f,r}$ over the maneuver to determine if the vehicle will rollover or skid. If either of these values rise above a predetermined threshold, the avoidance maneuver is performed. Otherwise, the vehicle continues driving and recalculates.

5.4.2 Simulation Results

The simulation results of the scenario described above are presented in this section for a full range of speeds and intervention distances. For the wheel lift threshold, the value of y_{zmp} was normalized as in Chapter 4 such that values above 1 or below -1 indicate wheel lift. For the tire skid threshold, tire slip angles above 10 deg or below -10 deg (front or rear) indicate skidding. Tire skid is important to consider in this scenario for several reasons. First, skidding is a dangerous situation that leads to loss of yaw stability in an emergency situation. Second, tire skid indicates the presence of significant nonlinearities that reduce the validity of a linear analysis. Falcone et al. also showed that striving to minimize tire slip angle improved controller performance by keeping the tires away from nonlinear regions [17]. The tire skid threshold of 10 deg was found in previous work [48] and has also been normalized about this point.

The maximum values of y_{zmp} and $\alpha_{f,r}$ over the test suite of speeds and intervention distances are shown in Fig. 5.5. The circles on the plots indicate whether the limiting intervention distance was dictated by wheel lift or tire skid for a particular speed. For example, the 13.4 m/s curve of Fig. 5.5 is circled on the y_{zmp} plot. This means that



Figure 5.5. Minimum intervention distances where wheel lift and tire skid occur for a flat road over various speeds. The circles indicate whether the limiting intervention distance was dictated by wheel lift (top) or tire skid (bottom) for a particular speed.

a greater intervention distance was needed to mitigate the threat of wheel lift at this speed. The results show that wheel lift precedes skidding at low speeds, while skidding precedes wheel lift at high speeds. This is an intuitive relationship; at lower speeds, the suspension dynamics have time to evolve over the longer lane change, resulting in more dominate rollover effects. The plots also show that, overall, higher speeds require a greater intervention distance for a safe maneuver.

It should be noted that the intervention distances determined here correspond with the last possible instant before the vehicle must intervene, i.e. maneuvers performed at the distances circled on the plots result in y_{zmp} and $\alpha_{f,r}$ values that rise to the brink of their thresholds. Ideally, the vehicle should not intervene before it is absolutely necessary. In practice, however, these thresholds could be more conservative, for example, at a predetermined percentage of the y_{zmp} and $\alpha_{f,r}$ normalized values. This would result in



Figure 5.6. Collision avoidance scenario: a moving vehicle approaching a stationary object on a banked curve. This figure illustrates a positive bank angle, while a negative bank angle slopes the opposite direction.

greater intervention distances and a less severe avoidance maneuver.

5.5 Steering Intervention for Collision Avoidance on a Banked Curve

5.5.1 Simulation Overview

In order to investigate scenarios in which the vehicle would experience an even greater rollover threat, simulations were performed for the avoidance maneuver on a banked curve. Figure 5.6 illustrates this scenario, where the lane change maneuver is performed up the slope of the terrain for a positive bank angle. Conversely, the lane change maneuver is performed down the slope of the terrain for a negative bank angle. All simulation procedures and parameters remain the same as those for the flat road, with the exception of a constant road superelevation now present throughout the maneuver.

Four different bank angles were simulated: a moderate $\pm 4.5 \text{ deg slope}$ (8% superelevation) and a severe $\pm 8 \text{ deg slope}$ (14% superelevation). These values are representative

Table 5.1. Road friction values used for different vehicle speeds based on highway design guidelines [57].

U	μ_{road}
13.4 m/s (30 mph)	0.20
20.1 m/s (45 mph)	0.16
26.8 m/s (60 mph)	0.13
33.5 m/s (75 mph)	0.10

of the terrain angles typically seen on highway curves [57]. The radius of the highway curve was determined for each speed using acceptable lateral acceleration values specified in highway design guidelines [57] according the the following:

$$R_{curve} = \frac{U^2}{g\left(\frac{\phi_t}{100} + \mu_{road}\right)} \sec\left(\arctan\left(\frac{\phi_t}{100}\right)\right)$$
(5.6)

In Eq. 5.6, ϕ_t is specified in percent superelevation and values of μ_{road} , the road friction, were used corresponding to the appropriate speed in [57] (Table 5.1).

Using knowledge of the vehicle's speed, the radius of the curve, and the bank angle of the curve, initial steering biases needed to navigate the curve (without a lane change) were determined. The open-loop iterative approach of Section 5.3 was repeated with the addition of terrain angle, resulting in a new mapping from Eq. 5.3 to the following:

$$\delta_f = A\sin(\omega t) + b_s \tag{5.7}$$

where b_s is the initial steering bias.

5.5.2 Simulation Results

The simulation results for the ± 4.5 deg slope (8% superelevation) and the ± 8 deg slope (14% superelevation) are shown in Fig. 5.7 and Fig. 5.8 respectively. Overall, the trends appear as expected; the introduction of terrain increases the minimum intervention dis-

tance needed for a safe maneuver, with higher speeds requiring more distance. The individual relationships between each scenario are examined in Section 5.6.

It should be noted that for all of the simulations involving road superelevation, wheel lift is the dominate threat and dictates the intervention distance (indicated by the circled curves) for most of the speeds. This is an intuitive relationship, as the addition of terrain increases the vehicle's rollover propensity. The only instance where the tire skid threshold is breached first is at high speeds (75 mph) with a positive bank angle (both the 4.5 deg and 8 deg cases). Skidding precedes rollover in this scenario because the vehicle is rapidly steering up the slope of the terrain, where gravity impedes the vehicle's motion.



Figure 5.7. Minimum intervention distances where wheel lift and tire skid occur for a banked curve (± 4.5 deg) over various speeds. The circles indicate whether the limiting intervention distance was dictated by wheel lift (top) or tire skid (bottom) for a particular speed.



Figure 5.8. Minimum intervention distances where wheel lift and tire skid occur for a banked curve ($\pm 8 \text{ deg}$) over various speeds. The circles indicate whether the limiting intervention distance was dictated by wheel lift (top) or tire skid (bottom) for a particular speed.

5.6 Comparison of Intervention Distances

This section compares the minimum intervention distances found for all scenarios described above. For each simulation, the minimum intervention distance, whether dictated by wheel lift or tire skid, was recorded as a function of speed. In other words, the absolute minimum distance needed is the distance for each speed where the wheel lift threshold or tire skid threshold is breached, whichever occurs first.

Figure 5.9 shows the results of each of the tested scenarios. The curve labeled "Rollover not considered" is the steering intervention result of [20] discussed in Section 5.2. The reader should keep in mind that the results of [20] are an idealized intervention distance, where the vehicle is assumed to achieve instantaneous lateral acceleration



Figure 5.9. Comparison of the minimum intervention distances needed for an open-loop lane change avoidance maneuver.

and factors such as rollover are ignored. Thus, the findings from the simulations of the previous sections clearly show the need for consideration of these factors in an emergency situation with a rollover-prone vehicle. In particular, when wheel lift and tire skid are considered, there is a considerable increase of approximately 150% in the necessary intervention distance.

Further insight can be gained by comparing the minimum intervention distances of the individual simulations. It can be seen that, in general, the avoidance maneuver requires greater intervention distance when performed on negative bank angles, especially at speeds above 20.1 m/s (45 mph). A high rollover threat occurs in this scenario when the vehicle steers down the slope to avoid the obstacle, and subsequently steers up the slope to remain within the opposite lane, effectively amplifying the roll dynamics. Another intuitive relationship seen in Fig. 5.9 is that large bank angles (± 8 deg) require more distance than the moderate bank angles ($\pm 4.5 \text{ deg}$) of the same sign. Naturally, this is a result of the vehicle's increased rollover propensity on more severe terrain.

These results indicate that it is important to consider a vehicle's roll and tire dynamics when developing collision avoidance algorithms. This is especially true when a rollover-prone situation is present, such as with SUVs and heavy trucks, or when a vehicle is driving on uneven terrain. Knowledge of these situations and the necessary intervention distance can help the development of closed-loop architectures that account for this behavior when necessary. As mentioned previously, the open-loop framework presented in this chapter can also be modified to examine other intervention strategies, such as braking, or different emergency scenarios.

Chapter 6 Closed-Loop Collision Avoidance and Rollover Prevention

6.1 Introduction

This chapter extends the work of Chapters 4 and 5 through implementation of a closedloop controller. The controller in this chapter is designed to safely navigate the vehicle around an obstacle while also maintaining roll and yaw stability. This is the same scenario considered in Chapter 5, but with added feedback control.

The open-loop analyses of Chapters 4 and 5 were beneficial in determining the minimum preview horizon and minimum intervention distance, respectively. However, an open-loop implementation is also impractical for use on a real vehicle. Unknown disturbances, model uncertainties, etc. contribute to overall vehicle behavior that would not be accounted for with an open-loop controller. Therefore, it is necessary to develop a closed-loop controller using the knowledge gained from the open-loop analyses.

The remainder of this chapter is organized as follows: first, a Linear-Quadratic Regulator (LQR) with full state weighting is presented and analyzed. Next, a modified



Figure 6.1. Controller setup for path following using model reference state trajectories and full state feedback.

control law is presented utilizing LQR with output weighting for ZMP regulation. Finally, a study is performed on appropriate weighting values for ZMP regulation based on several safety criteria.

6.2 Linear-Quadratic Regulator with Full State Weighting

The first method for closed-loop control used the standard infinite horizon LQR algorithm with full state weighting, with controller gain determined by

$$\min_{u(t)} J = \int_0^\infty \left[\vec{x}(t)^T Q \vec{x}(t) + \vec{u}(t)^T R \vec{u}(t) \right] dt$$

s.t. $\dot{\vec{x}}(t) = A \vec{x}(t) + B \vec{u}(t)$ (6.1)

where Q is the state weighting matrix and R is the input weighting matrix.

In order to test the path following performance of this controller, the state feedback scheme was implemented according to Fig. 6.1. Here, the open-loop steering maneuvers of Chapter 5 were used to develop model reference trajectories for all the vehicle states. The controller then operated off of the error between the model reference states and the vehicle states.

Simulations of this controller were performed using the standard roll model with tire lag using vehicle parameters provided in Table 3.3, and Q and R matrices chosen as such:

$$Q = \text{diag}[50 \quad 0.001 \quad 10 \quad 0.1 \quad 0.1 \quad 0 \quad 10 \quad] \tag{6.2}$$

$$R = 1 \tag{6.3}$$

corresponding to the state vector for the roll model with tire lag of:

J

$$\vec{x} = \begin{bmatrix} y & V & r & \dot{\phi}_r & \phi_r & F_f & F_r & \psi \end{bmatrix}^T$$
(6.4)

The values of the state weighting matrix, Q, were chosen to emphasize the tracking of three states in particular: y, r, and ψ . A relatively high weight on y (50) demands that the vehicle follow the lateral position trajectory specified by the reference model. Tracking of r and ψ with moderately high weights (10), meanwhile, produces a smooth steering action without sacrificing lateral position tracking.

Additionally, the simulations assumed a flat road and a vehicle speed of 20.1 m/s (45



Figure 6.2. Lateral position trajectory of a lane change avoidance maneuver using LQR full state weighting following model reference states. The maneuver was performed at 20.1 m/s (45 mph) over 29.7 m of longitudinal travel (the minimum intervention distance at this speed).
mph) due to the roll-before-skid behavior seen at this speed in Chapter 5. The steering input was rate limited to 4 rad/s to mimic the vehicle's steering actuator dynamics [60] and saturated at 0.4 rad and -0.4 rad to simulate the limits of the steering rack. Figure 6.2 shows the path following performance of this controller at 45 mph for a maneuver performed at the the minimum intervention distance (Fig. 5.9 of Chapter 5).

Figure 6.2 demonstrates that the controller is able to maintain excellent path following of the open-loop reference trajectory. Concerns arise, however, when examining the values of y_{zmp} and the tire slip angles over the course of the maneuver, as shown in Fig. 6.3. As shown by the plot of y_{zmp} for the maneuver, the controller slightly crosses over the wheel lift threshold. This is a natural result of a closed-loop tracker. There



Figure 6.3. y_{zmp} and tire slip angles for full state weighting LQR over the maneuver shown in Fig. 6.2.

will always be some lag (for non-preview solutions) between the reference trajectory and model output, as the controller only reacts to the growth of an error term. One must also keep in mind that the reference trajectory provided to the vehicle constitutes the absolute minimum intervention distance needed before wheel lift, excessive tire skid, or a collision occurs. Thus, because the vehicle is not able to follow the reference trajectory exactly, the value of y_{zmp} breaches the safety threshold.

6.3 Linear-Quadratic Regulator with Output Weighting

The results of Section 6.2 show the need for a solution that actively monitors the vehicle's roll threat. The computation of the intervention distance used in this research is still an "offline" exercise, completely model-based. In this sense, if the real vehicle parameters differ from the ones used in the model, the vehicle runs the risk of underestimating the intervention distance. This would result in a controller that accurately tracks an *unsafe* trajectory. The feedback scheme of Section 6.2 has no way of modifying the maneuver, even if y_{zmp} is approaching a critical level.

Thus, the fact that y_{zmp} can be added as an output of the system served as motivation to change the state regulator problem of Eq. 6.1 to the output regulator problem of the following:

$$\min_{u(t)} J = \int_0^\infty \left[\vec{y}(t)^T \widetilde{Q} \vec{y}(t) + \vec{u}(t)^T R \vec{u}(t) \right] dt$$

s.t. $\dot{\vec{x}}(t) = A \vec{x}(t) + B \vec{u}(t)$
 $\vec{y}(t) = C \vec{x}(t) + D \vec{u}(t)$ (6.5)

where \tilde{Q} is the updated weighting matrix for an output weighting framework. For this study, the output of the system includes all of the state variables of Eq. 6.4, plus two additional values: y_{zmp} and $y_{zmp}(t+T)$ such that

$$\vec{y}(t) = \begin{bmatrix} \vec{x}(t) & y_{zmp} & y_{zmp}(t+T) \end{bmatrix}^T$$
(6.6)

Subsequently, the updated state weighting matrix \tilde{Q} is identical to the LQR Q matrix through its weights on the states, but now two additional terms are added along the diagonal: Q_{yzmp} and $Q_{yzmp(t+T)}$. These terms represent the importance of keeping y_{zmp} and $y_{zmp}(t+T)$ close to zero, respectively. High values of Q_{yzmp} and $Q_{yzmp(t+T)}$ will naturally indicate that minimizing y_{zmp} and $y_{zmp}(t+T)$ is a higher priority than state tracking, while small values prioritize good state tracking, even if this action produces high ZMP values.

In order to apply full state feedback with LQR output weighting, the integrand of the performance index of Eq. 6.5 must be rewritten as a function of the state vector. This is because the LQR algorithm relies on the assumption of full state feedback for the calculation of the optimal gain matrix, even when the weighting is placed on the output vector. Substituting the equation for \vec{y} into the integrand and rearranging terms produces the following:

$$(C\vec{x} + D\vec{u})^T \tilde{Q}(C\vec{x} + D\vec{u}) + \vec{u}^T R\vec{u}$$
(6.7)

$$(\vec{x}^T C^T + \vec{u}^T D^T) \widetilde{Q} (C\vec{x} + D\vec{u}) + \vec{u}^T R\vec{u}$$
(6.8)

$$\vec{x}^T C^T \tilde{Q} C \vec{x} + 2\vec{u}^T D^T \tilde{Q} C \vec{x} + \vec{u}^T D^T \tilde{Q} D \vec{u} + \vec{u}^T R \vec{u}$$
(6.9)

$$\vec{x}^T [C^T \widetilde{Q} C] \vec{x} + 2\vec{x}^T [C^T \widetilde{Q} D] \vec{u} + \vec{u}^T [D^T \widetilde{Q} D + R] \vec{u}$$
(6.10)

Letting the terms $C^T \tilde{Q} D = \tilde{N}$ and $D^T \tilde{Q} D + R = \tilde{R}$ we arrive at the following:

$$\vec{x}^T [C^T \tilde{Q} C] \vec{x} + 2\vec{x}^T \tilde{N} \vec{u} + \vec{u}^T \tilde{R} \vec{u}$$
(6.11)

Preliminary results showed best performance by setting \tilde{N} to zero when solving for the optimal gains in the LQR output weighting formulation. Thus, the final optimization problem for full state feedback LQR with output weighting is written as such:

$$\min_{u(t)} J = \int_0^\infty \left[\vec{x}(t)^T C^T \widetilde{Q} C \vec{x}(t) + \vec{u}(t)^T \widetilde{R} \vec{u}(t) \right] dt$$

s.t. $\dot{\vec{x}}(t) = A \vec{x}(t) + B \vec{u}(t)$
 $\vec{y}(t) = C \vec{x}(t) + D \vec{u}(t)$ (6.12)

The updated input weight matrix, \tilde{R} , takes the weights from the standard LQR input weighting, R, and adds a semi-definite positive "correction" term. This term accounts for the fact that the throughput matrix D has nonzero terms in the calculation of y_{zmp} and $y_{zmp}(t+T)$: the direct effect of steering input on ZMP. The reader should also note that the "correction" term is proportional to the tuning parameters Q_{yzmp} and $Q_{yzmp(t+T)}$. The expected effect is that larger values of these weights will produce a larger \tilde{R} , hence smaller steering inputs to the system, as compared to the standard LQR of Section 6.2.

Additionally, if Q_{yzmp} and $Q_{yzmp(t+T)}$ are assigned weights of zero, the output regulator is reduced to the state regulator of Section 6.2. Thus, the controller is able to perform trajectory following under normal driving scenarios without retuning the weights on the states. However, if the vehicle detects that it is in an emergency situation and may experience wheel lift, the weights on Q_{yzmp} and $Q_{yzmp(t+T)}$ can be adjusted for a safe maneuver. The results of the following sections aim to determine how best to use these weights for safe path following in the collision avoidance scenario of Chapter 5.

6.3.1 Weight Tuning for Q_{yzmp}

To illustrate the effect of ZMP weighting, the avoidance maneuver of Section 6.2 was simulated with the LQR output regulator of Eq. 6.12. The weights on the states remain the same as in Section 6.2, specified in Eq. 6.2. Figure 6.4 shows the path following performance of the vehicle with $Q_{yzmp} = 100$ and $Q_{yzmp}(t+T) = 0$, while Fig. 6.5 shows the behavior of y_{zmp} and tire slip angles. These plots exhibit the expected trends of increasing the weight on y_{zmp} regulation: in order to reduce the value of y_{zmp} (Fig. 6.5), the controller sacrifices state tracking, resulting in overshoot of the reference trajectory (Fig. 6.4). In other words, reducing the value of y_{zmp} requires the controller to command smaller changes to the steering input.

Further insight into these trends can be gained by examining the contribution of ZMP regulation to the overall steering input. Thus, the control structure was broken up



Figure 6.4. Lateral position trajectory of a lane change avoidance maneuver using LQR output weighting on y_{zmp} . The maneuver was performed at 20.1 m/s (45 mph) over 29.7 m of longitudinal travel (the minimum intervention distance at this speed).



Figure 6.5. y_{zmp} and tire slip angles for LQR output weighting over the maneuver shown in Fig. 6.4.

according to Fig. 6.6. In this structure, δ_{zmp} represents the steering input commanded to keep y_{zmp} at zero, while δ_{state} represents the steering input commanded to maintain state tracking. Figure 6.7 shows each of these steering inputs over the avoidance maneuver of Fig. 6.4, as well as the combined steering input, δ_{CL} . This plot explicitly illustrates that the steering input required for state tracking and the steering input required for ZMP regulation have opposite objectives. The net effect on the combined steering, δ_{CL} , is a less severe steering input when compared to $Q_{yzmp} = 0$. The reader should note that controller saturation and rate limiting are only applied to the overall input, δ_{CL} . Thus, the high peaks of δ_{state} and δ_{zmp} can be intuitively explained as such: in order to perform state tracking, δ_{state} grows. ZMP regulation through δ_{zmp} , however, desires



Figure 6.6. Controller setup used to examine the ZMP regulator contribution towards controller steering input.



Figure 6.7. Steering angle of the controller for different weightings of Q_{yzmp} . The steering contributions for ZMP regulation and state tracking are also shown.

smaller steering inputs and cancels some of the steering commanded through δ_{state} , resulting in a reduced overall steering input. This in turn requires δ_{state} to grow even more and so on. The net effect is an LQR output regulator that tracks the reference trajectory less aggressively due to the opposing objective of keeping steering changes small for ZMP regulation. This "effect of ZMP regulation" can be thought of simply as the difference in δ_{CL} for $Q_{yzmp} = 0$ and $Q_{yzmp} = 100$, shown in Fig. 6.8.

In order to examine how the magnitude of Q_{yzmp} affects the avoidance maneuver,



Figure 6.8. The combined steering angle of the controller, δ_{CL} , for different weightings of Q_{yzmp} . Subtracting the curve for $Q_{yzmp} = 100$ from the curve for $Q_{yzmp} = 0$ results in the steering effect of ZMP weighting.

simulations were performed over a range of Q_{yzmp} values. The effectiveness of the resulting maneuver was evaluated by four criteria: the maximum lateral position, y_{max} (indicating whether the vehicle stays on the road), the lateral position when the obstacle is reached, y_{obs} (indicating whether the vehicle avoids the collision), the maximum magnitude of y_{zmp} , $y_{zmp,max}$ (indicating the vehicle's wheel lift threat), and the maximum magnitude of tire slip angle, α_{max} (indicating the vehicle's proximity to skidding). The ideal value of y_{max} should be 3.65 m, meaning there is no overshoot of the center of the lane. However, the full width of the lane can be utilized as long as the tires remain within the bounds of the lane, i.e. $y_{max} \leq 4.67$ m. Additionally, the ideal value of y_{obs} should be 3.65 m, meaning the vehicle is centered in the opposite lane when the obstacle is reached. Once again, however, if the full width of the lane is utilized, $y_{obs} \geq 2.64$ m results in a successful maneuver out of the blocked lane. The thresholds for $y_{zmp,max}$ and α_{max} remain the same as in Chapters 4 and 5 after being normalized.



Figure 6.9. Effect of Q_{yzmp} weight on four safety criteria for a lane change avoidance maneuver. For each speed, the maneuver was performed at the minimum intervention distance of Fig. 5.9. The value of $Q_{yzmp(t+T)}$ was held at zero.

For the simulations, the avoidance maneuvers were performed at the minimum intervention distance for a flat road as a function of the vehicle's speed, given by Fig. 5.9 of Chapter 5. Figure 6.9 shows the effect of Q_{yzmp} on the four safety criteria discussed above while holding $Q_{yzmp(t+T)}$ at zero. It is evident that there is a trade-off between $y_{zmp,max}$, α_{max} and y_{max} , y_{obs} . Larger values of Q_{yzmp} tend to reduce the values of $y_{zmp,max}$ and α_{max} by demanding a less aggressive maneuver. The smaller changes in steering input, however, result in greater overshoot of y_{max} and smaller values of y_{obs} .

Figure 6.9 also shows that the performance of the LQR output regulator is speed dependent, especially when considering the vehicle's rollover threat. For all speeds, values of Q_{yzmp} roughly below 50 fail to effectively reduce the rollover and/or tire skid threat. Values above 100, meanwhile, bring the vehicle into the safe operating zones for all four criteria. Additionally, the curves for all four criteria exhibit asymptotic behavior as the value of Q_{yzmp} is increased; thus, a gain scheduling approach based on the vehicle speed can be adopted. For the scenario shown here, Q_{yzmp} values between 100 and 500 seem to be feasible solutions, depending on whether the controller is designed to favor roll/skid robustness or path following robustness.

6.3.2 Bank Angle Effect on Q_{yzmp}

This section examines the effect of bank angle (road superelevation) on the performance of the LQR output regulator. The simulation parameters and procedure remain the same, however, instead of a flat road, the vehicle is set to experience a -8 deg (14% superelevation) road bank angle, as in Chapter 5. This value is representative of a severe terrain angle typically seen on highway curves [57] and was shown in Chapter 5 to produce the greatest rollover threat. Figure 6.10 shows the controller performance on the -8 deg bank angle while varying Q_{yzmp} . It should be noted that the maneuvers of Fig. 6.10 were performed at the minimum intervention distances of Fig. 5.9 for a -8



Figure 6.10. Effect of Q_{yzmp} weight on four safety criteria for a lane change avoidance maneuver on a -8 deg (14% superelevation) bank angle. For each speed, the maneuver was performed at the minimum intervention distance of Fig. 5.9 for a -8 deg bank angle. The value of $Q_{yzmp(t+T)}$ was held at zero.

deg bank angle, not a flat road. Because of the increased intervention distance resulting from the terrain, Fig. 6.10 shows similar trends to those of Fig. 6.9 for all four safety criteria, with slightly increased values of y_{zmp} and greater overshoot in y_{max} . The results show that even with the introduction of a -8 deg bank angle, values of Q_{yzmp} between approximately 100 and 500 seem to be the best solution for balancing the trade-offs between the four safety criteria.

During an emergency scenario, situations may arise when the vehicle is not able to perform an avoidance maneuver at the minimum intervention distance. For example, an animal unexpectedly coming into the vehicle path may necessitate that an avoidance maneuver be performed at shorter intervention distances. For this reason, the controller performance was also evaluated for a -8 deg road bank angle at the minimum intervention distances of Fig. 5.9 specified for a *flat* road. Figure 6.11 shows the results of this analysis. It can be seen that the vehicle experiences higher values of y_{zmp} over the more severe maneuver; however, it is still able to follow a safe avoidance trajectory, particularly around a Q_{yzmp} value of 300. Thus, regulating ZMP as part of the closed-loop control is able to prevent wheel lift even under severe maneuvers and terrain variations.

6.3.3 Weight Tuning for $Q_{yzmp(t+T)}$

This section now examines the effect of regulating the previewed value of ZMP, $y_{zmp}(t + T)$, in the LQR output weighting controller. As in the previous section, the avoidance maneuver of Section 6.2 was simulated with the LQR output regulator and state weights specified in Eq. 6.2. However, the implementation of $y_{zmp}(t + T)$ now presents an additional simulation variable: the preview horizon length, T. The choice of the preview horizon was the subject of study in Chapter 4 and the results are utilized here. In particular, the collision avoidance maneuver very closely resembles the steering trajectory of corrective steering maneuver #2. Thus, the preview time was chosen based on the



Figure 6.11. Effect of Q_{yzmp} weight on four safety criteria for a lane change avoidance maneuver on a -8 deg (14% superelevation) bank angle. For each speed, the maneuver was performed at the minimum intervention distance of Fig. 5.9 for a *flat* road. The value of $Q_{yzmp(t+T)}$ was held at zero.



Figure 6.12. Lateral position trajectory of a lane change avoidance maneuver using LQR output weighting on $y_{zmp}(t+T)$. The maneuver was performed at 20.1 m/s (45 mph) over 29.7 m of longitudinal travel (the minimum intervention distance at this speed).

avoidance maneuver steering frequency and magnitude according to Fig. 4.12.

Figure 6.12 shows the path following performance of the vehicle with $Q_{yzmp} = 0$ and $Q_{yzmp}(t+T) = 100$, while Fig. 6.13 shows the behavior of y_{zmp} and tire slip angles. The lateral position trajectory of Fig. 6.12 exhibits similar trends to those of Fig. 6.4, namely overshoot of the reference trajectory. Figure 6.13, however, illustrates some unexpected results. In the plot of ZMP, it can be seen that weighting $Q_{yzmp(t+T)}$ produces the desired result of decreasing the peak values of $y_{zmp}(t+T)$. However, weighting on $Q_{yzmp(t+T)}$ actually increases the peak values of y_{zmp} over the maneuver. Thus, in striving to regulate the previewed solution of ZMP, the vehicle follows a trajectory with an even greater rollover threat. This shows that steering inputs designed to reduce the vehicle's rollover threat in the near future do not necessarily result in a safe maneuver at the present time.

This trend is further illustrated by Fig. 6.14, where y_{zmp} and $y_{zmp}(t+T)$ have been plotted for several weighting combinations of Q_{yzmp} and $Q_{yzmp(t+T)}$: (a) $Q_{yzmp} = 0$ and



Figure 6.13. y_{zmp} and tire slip angles for LQR output weighting over the maneuver shown in Fig. 6.12.

 $Q_{yzmp(t+T)} = 0$, (b) $Q_{yzmp} = 100$ and $Q_{yzmp(t+T)} = 0$, (c) $Q_{yzmp} = 0$ and $Q_{yzmp(t+T)} = 100$, (d) $Q_{yzmp} = 100$ and $Q_{yzmp(t+T)} = 100$. Additionally, Fig. 6.14 compares the peak values of y_{zmp} and $y_{zmp}(t+T)$ for each weighting combination. The simulation maneuver and parameters were identical to those of Fig. 6.12, with the exception of Q_{yzmp} and $Q_{yzmp(t+T)}$. The results agree with the observations seen in Fig. 6.13. When compared to zero weighting of ZMP (Fig. 6.14(a)), sole weighting of Q_{yzmp} offers the most effective means of rollover prevention by reducing the peak values of both y_{zmp} and $y_{zmp}(t+T)$ (Fig. 6.14(b)). Sole weighting of $Q_{yzmp(t+T)}$, meanwhile, reduces the peak value of $y_{zmp}(t+T)$, but increases the peak value of y_{zmp} (Fig. 6.14(c)), effectively sacrificing current performance to mitigate the future rollover threat. Similarly, weighting of both Q_{yzmp} and $Q_{yzmp(t+T)}$ also reduces the peak value of $y_{zmp}(t+T)$, but increases the



Figure 6.14. y_{zmp} and $y_{zmp}(t+T)$ over the maneuver shown in Fig. 6.12 for various weightings of Q_{yzmp} and $Q_{yzmp(t+T)}$. The largest magnitude of y_{zmp} and $y_{zmp}(t+T)$ over the maneuver is shown to the right of the corresponding plot.

peak value of y_{zmp} (Fig. 6.14(d)). This last result is the most interesting; while the introduction of Q_{yzmp} weighting in Fig. 6.14(b) provides an effective means of reducing the vehicle's rollover threat, the addition of $Q_{yzmp(t+T)}$ weighting to this scenario in Fig. 6.14(d) increases the peak values of both y_{zmp} and $y_{zmp}(t+T)$. Thus, weighting of $Q_{yzmp(t+T)}$ increases the vehicle's rollover threat even when Q_{yzmp} is weighted as well.

Once again, simulations were performed over a range of speeds and values of $Q_{yzmp(t+T)}$ to examine the effect of $Q_{yzmp(t+T)}$ weight on the four safety criteria established for the avoidance maneuver. Figure 6.15 provides the results of this analysis while holding



Figure 6.15. Effect of $Q_{yzmp(t+T)}$ weight on four safety criteria for a lane change avoidance maneuver. For each speed, the maneuver was performed at the minimum intervention distance of Fig. 5.9. The value of Q_{yzmp} was held at zero.

 Q_{yzmp} at zero. As expected, weighting of $Q_{yzmp(t+T)}$ is not as effective as weighting of Q_{yzmp} in preventing wheel lift during the avoidance maneuver, and actually increases the rollover threat for certain values ($Q_{yzmp(t+T)} < 100$). Values of $Q_{yzmp(t+T)}$ greater than approximately 500 result in a safe maneuver for all four criteria; however, the fact remains that even in this range, weighting of Q_{yzmp} is more effective in reducing the vehicle's rollover threat.

These results lead to several conclusions. It can be seen that simply including $y_{zmp}(t+T)$ in the control optimization does not necessarily decrease the vehicle's rollover threat; in fact, under some circumstances and improper weighting of $Q_{yzmp(t+T)}$, this action results in an increased rollover threat. Figure 6.14(d) even shows that introducing $Q_{yzmp(t+T)}$ weighting in addition to Q_{yzmp} weighting results in an increased rollover threat. Therefore, while an exhaustive search of optimal Q_{yzmp} and $Q_{yzmp(t+T)}$ combinations was not conducted, the results indicate that best performance is achieved through LQR output regulation that only regulates y_{zmp} . Subsequently, this result also leads to the conclusion that, while $y_{zmp}(t+T)$ provides useful information about the vehicle's rollover threat in the near future, it may be prudent not to include it in the controller optimization policy. The resulting controller could then utilize information about $y_{zmp}(t+T)$ as an indicator for when a new control policy (one that strives to reduce the rollover threat) should be used. Put another way, dangerous values of $y_{zmp}(t+T)$ could be used to return the vehicle's controller by increasing the weight on Q_{yzmp} . This would result in a controller that only sacrifices state tracking performance when a severe rollover threat is predicted.

Chapter 7 Conclusions & Future Work

This chapter summarizes the important conclusions found in this thesis. Additionally, possibilities for future work are discussed based on the most promising avenues of this research.

7.1 Conclusions

The work in this thesis covered several aspects of vehicle rollover prevention. Chapter 4 aimed to implement a previewed solution of the zero-moment point metric as an indicator of future rollover threats. In particular, the minimum preview horizon needed to predict and prevent wheel lift was investigated through open-loop steering maneuvers representative of a driver in an emergency situation. Results showed that preview horizons between 0.1 s and 0.7 s proved capable of preventing wheel lift through corrective intervention. The reader should note, however, that these results were obtained for one particular scenario involving a rollover-prone vehicle and rollover-inducing driving conditions. Also, changes to the intervention strategy (i.e. braking instead of steering) would likely change the range of necessary preview horizons. Nevertheless, the methods presented in this thesis can be readily modified to study changes to the simulation param-

eters or intervention strategy. The choice of an effective preview horizon for predictive control-based solutions remains an open research problem. A preview horizon that is too short may not provide enough information, while one that is too long is computationally expensive and cannot perform in real-time. The results presented in Chapter 4 provide a good baseline when choosing a preview horizon in the design of closed-loop model predictive control (MPC) approaches for rollover prevention.

This thesis also addressed strategies to mitigate a vehicle's rollover threat in a collision avoidance scenario. Chapter 5 studied the question of the minimum intervention distance needed to avoid a stationary object, while also preventing wheel lift and tire skid. Results from this analysis showed that when roll and tire dynamics are considered, there is approximately a 150% increase in the intervention distance needed for a safe maneuver when compared to the idealized avoidance scenario of [20]. This indicates the importance of considering a vehicle's rollover threat when developing optimized avoidance trajectories, especially for rollover-prone vehicle such as trucks and SUVs. The results of this analysis also showed that more intervention distance is needed when the avoidance maneuver is performed on banked roads, an effect which amplifies the vehicle's rollover propensity. This intuitive result reminds readers that terrain effects, information which is attainable through mapped roadways, should also be factored into the design of avoidance trajectories.

The results of Chapters 4 and 5 were then utilized in Chapter 6 in the design of a closed-loop controller for collision avoidance and rollover prevention. Preliminary results of the standard LQR framework for state tracking showed excellent path following, but an unreliable means of preventing wheel lift. In essence, the standard LQR controller did not actively strive to reduce the vehicle's rollover threat. Thus, an LQR output weighting framework was chosen to actively regulate y_{zmp} and $y_{zmp}(t+T)$ in addition to state tracking. Using this method, explicit tuning parameters were introduced that

represent the relative importance of rollover prevention with respect to state tracking. An analysis was then performed to determine appropriate weights for these terms: Q_{yzmp} for y_{zmp} and $Q_{yzmp(t+T)}$ for $y_{zmp(t+T)}$. Results showed that rollover prevention through active weighting of y_{zmp} and state tracking are competing objectives. ZMP regulation was indeed able to reduce the vehicle's overall rollover threat even in the presence of terrain variations, but at the cost of path following performance. Ultimately, this is a result of the need for smaller changes in steering input to reduce the rollover threat. The analysis then showed that values of Q_{yzmp} between 100 and 500 are the most effective at balancing the trade-off between rollover prevention and state tracking. A second result of this analysis indicated that actively regulating the previewed solution of ZMP through $Q_{yzmp(t+T)}$ is not an effective means of reducing the vehicle's real-time rollover threat. In fact, if $Q_{yzmp(t+T)}$ is weighted incorrectly, it was shown that the vehicle follows a trajectory that actually increases the real-time rollover threat. Thus, while information about about the vehicle's future rollover threat is useful for warning purposes, it should not be actively regulated for control.

7.2 Future Work

The research presented in this thesis offers several pertinent avenues of future work. Model predictive control remains one of the most promising approaches to vehicle control, especially in emergency situations where significant nonlinearities are imposed on the system. Therefore, employing the results of this thesis in an MPC framework has the potential to offer a very effective means of optimal rollover prevention and path following. As discussed in Chapter 2, MPC produces an optimal sequence of control inputs up to a user-specified control horizon. This sequence of control inputs is iteratively calculated at each time step and can be subject to performance constraints. While the LQR output regulator of Chapter 6 proved to be an effective means of reducing the vehicle's rollover threat, the vehicle is not necessarily following an optimal avoidance trajectory. Thus, an MPC approach could be used to develop an optimal avoidance trajectory and navigate the vehicle along it. Additionally, the calculation of this avoidance trajectory could be subject to constraints y_{zmp} , i.e. the trajectory must not induce wheel lift.

Another potential avenue of future work would be to investigate the feasibility of implementing a switching law on the LQR output regulator presented in Chapter 6. When the control weights placed on y_{zmp} and $y_{zmp}(t+T)$ are zero, the output regulator reduces to the standard LQR controller used for state tracking. This is useful for tracking trajectories in non-emergency situations, where rollover is not a critical concern. A switching law would allow the controller to modify the gain of ZMP based on the vehicle's current and future rollover threat. However, due to the possibility of switching instabilities, a control scheme such as this would need to be heavily studied. Similarly, a feedforward term could be added from the open-loop, driver steering input. This would result in a controller that offers control of the vehicle to the driver, but then makes minor corrections when the driver commands a dangerous steering input.

Yet another area of future work would be to test different intervention strategies. While this thesis focused strictly on high-speed steering intervention, these studies should be repeated to determine the effect of alternative interventions such as braking or mixed steering and braking. The most effective intervention strategy is highly dependent on the scenario in question; low speeds, moving obstacles, different vehicle configurations, threat level, etc. can all affect the decision of the optimal policy. Subsequently, results such as the necessary preview horizon and minimum intervention distance would likely change given a new intervention strategy.

Finally, the algorithms used in this thesis and their corresponding results would greatly benefit from full-scale vehicle testing. The 1989 GMC 2500 pick-up truck used in the simulations has previously been automated for vehicle testing at Penn State. Experimental testing is essential in validating the techniques and results of this thesis, especially when considering the safety-critical nature of the work. These tests would undoubtedly reveal areas for algorithm refinement, as well as unforeseen problems that cannot be addressed in simulation.

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